

ALGEBRAIC GEOMETRY

HOMEWORK 1

Due Tu 15.02.04

- (1) Find all points on the following curves with coordinates in the finite fields \mathbb{F}_4 and \mathbb{F}_5 :
 - the line $x - 2y + 1 = 0$;
 - the 'circle' $x^2 + y^2 + 1 = 0$.
- (2) Determine the rational points on the hyperbola $X^2 - 3Y^2 = 1$ with as many methods as possible.
- (3) Show that the curve $X^{1/3} + Y^{1/3} = Z^{1/3}$ is a plane algebraic curve.
- (4) Use the sweeping line technique to parametrize the conic $x^2 - y^2 + 2x + 1 = 0$ using
 - (a) $P = (0, 1)$
 - (b) $Q = (-1, 0)$as your starting point. Explain your observations (if you can't, use sing surf to sketch the curve).