(1) Find all points on the following curves with coordinates in the finite fields \( F_4 \) and \( F_5 \):
- the line \( x - 2y + 1 = 0 \);
- the 'circle' \( x^2 + y^2 + 1 = 0 \).

(2) Determine the rational points on the hyperbola \( X^2 - 3Y^2 = 1 \) with as many methods as possible.

(3) Show that the curve \( X^{1/3} + Y^{1/3} = Z^{1/3} \) is a plane algebraic curve.

(4) Use the sweeping line technique to parametrize the conic \( x^2 - y^2 + 2x + 1 = 0 \) using
   (a) \( P = (0, 1) \)
   (b) \( Q = (-1, 0) \)
   as your starting point. Explain your observations (if you can’t, use sing surf to sketch the curve).