

## ALGEBRAIC GEOMETRY

### HOMEWORK 4

- (1) Compute the minimal polynomial of  $\sqrt{2} + \alpha$ , where  $\alpha$  is a root of the cubic polynomial  $x^3 + x + 1$ .
- (2) Compute the points of intersection and their multiplicities for the circle  $x^2 + y^2 - 2y = 0$  and the parabola  $y = x^2$ 
  - (a) directly;
  - (b) using resultants
- (3) Parametrize the quadric  $x^2 + y^2 + z^2 = 1$  using lines through  $(-1, 0, 0)$ . Which points are not covered by the parametrization? Is the projectivized parametrization bijective?
- (4) Show that the cubic surface  $X^2 + Y^3 - Y^2 + Z^2 = 0$  has a singular point. Parametrize the surface by using the pencil of lines through the origin.
- (5) Show that the cubic surface

$$X^2Y - X^2 + YZ^2 + Z^2 = 0$$

has a singular line, and find a parametrization.