

ALGEBRAIC GEOMETRY

PROBLEMS

- (1) Find all points on the projective closure of the curve $y^2 = x^3 + x$ over \mathbb{F}_3 .
- (2) Parametrize the conic $\mathcal{C} : x^2 + xy + y^2 = 3$ over \mathbb{Q} . Extend the corresponding map $\phi : \overline{A}^1\mathbb{Q} \rightarrow \mathcal{C}(\mathbb{Q})$ to a polynomial map $\phi^\# : \mathbb{P}^1\mathbb{Q} \rightarrow \mathcal{C}^\#(\mathbb{Q})$. Is ϕ injective, surjective, bijective? What about $\phi^\#$?
- (3) Find all singular points on $x^3 - y^2 - x^4 - y^4 = 0$.
- (4) Find all singular points on $x^3 + y^3 + 1 + 3axy = 0$, where $a \in \mathbb{C}$.
- (5) Compute the tangent of $x^3 + y^3 + 1 = 0$ at infinity.
- (6) Show that $X^3 + Y^3 = Z^4$ does not have nonconstant solutions in $\mathbb{C}[T]$.
- (7) Let C be a cubic with three double points. Show that C consists of three lines.