ALGEBRAIC GEOMETRY

MIDTERM 1, MARCH 22, 2004

(1) (5) Find all points on the projective closure of the curve $y^2 = x^3 + x$ over $\mathbb{F}_3$.

(2) (10) Find all singular points on $x^3 + y^3 + 1 + 3axy = 0$, where $a \in \mathbb{C}$.

(3) (20) Parametrize the conic $\mathcal{C} : x^2 + xy + y^2 = 3$ over $\mathbb{Q}$. Extend the corresponding map $\phi : \overline{A}^1\mathbb{Q} \longrightarrow \mathcal{C}(\mathbb{Q})$ to a polynomial map $\phi^{\#} : \mathbb{P}^1\mathbb{Q} \longrightarrow \mathcal{C}^{\#}(\mathbb{Q})$. Is ϕ injective, surjective, bijective? What about $\phi^{\#}$?

(4) (10) Compute the tangent of the real curve $x^3 + y^3 + 1 = 0$ at infinity.

(5) (15) Show that $X^3 + Y^3 = Z^4$ does not have nonconstant solutions in the polynomial ring $\mathbb{C}[T]$.

(6) (20) Let C be a cubic with three double points. Show that C consists of three lines. Are there cubics with exactly two double points?

(7) (20) Consider the curve $C : (x^2+y^2)^2 - x^2 + y^2 = 0$; for all lines L through $(0,0)$, compute the intersection multiplicity of C and L.