HISTORY OF MATHEMATICS

HOMEWORK 3

This is about Archimedes' computation of the area of a segment of a parabola.

(1) Proof Archimedes's result about the area of segments of a parabola using calculus: consider the parabola $y = ax^2$ and compute the area A between the chord defined by $R = (r, ar^2)$ and $S = (s, as^2)$ as well as the area B of the triangle RST, where $T = (\frac{1}{2}(r+s), \frac{a}{4}(r+s)^2)$, and show that $A = \frac{4}{3}B$.

(2) Compute the area A of a segment of the unit circle defined by the points $R = (-u, \sqrt{1-u^2})$ and $S = (u, \sqrt{1-u^2})$, and compare it to the area B of the triangle with maximal are inside this segment, namely RST with $T = (0, 1)$. Show that the ratio $A : B$ depends on u; also derive from your calculations that $\sin x \leq x$ for $x > 0$.

The Chinese had a formula for the area of a segment with base $s = 2u$ and height h: $A = \frac{1}{2}(sh + h^2)$. The same formula appears in a manuscript from Cairo written in the third century BC, and in the metrica of Heron of Alexandria. Develop your formula for A into a power series and show that $A = \frac{2}{3}sh + O(u^4)$, where $O(u^4)$ is a power series consisting of terms of order 4 and higher.

(3) We have seen that the triangle Archimedes inscribes into a segment of a parabola is the one with maximal area. Describe the quadrilateral with maximal area that can be inscribed into a segment of a parabola.

Extra credit: compute the ratio of the areas of the segment and the quadrilateral with maximal area.

(4) Reading assignment: read the three articles about zero on the web page.
- **Briefly** summarize the main claims made by the authors;
- Discuss whether the authors seem biased, and support your claims.