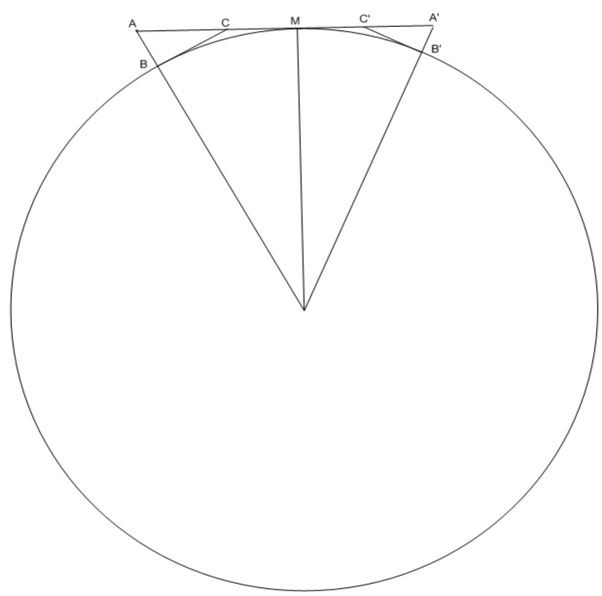
HISTORY OF MATHEMATICS

HOMEWORK 2

This is about Euclid's proof that circles are to each other as the squares of their diameters (Book XII, Proposition 2).

(1) Prove (similarly to what we did in class) that the area A_n of a 2^n-gon circumscribed about a circle with area A satisfies
$$A_n < (1 + \frac{1}{2^{n-2}})A.$$
(2) Complete the proof that $A_1 : A_2 = d_1^2 : d_2^2$ using the formula above.
(3) Study Euclid's original proof. Which parts of our proof occurs in Euclid?
(4) Did Euclid use circumscribed polygons to finish his proof? How exactly did he finish his proof?

Hints for proving the estimate in Problem 1.



In the figure above, AA' is a side of a 2^n-gon circumscribed about the circle. CC' is a side of a 2^{n+1}-gon, BC and $B'C'$ are half of such sides. Consider the segment ABM. You want to prove that segment$(BCM) < \frac{1}{2}$segment(ABM). To this end, compare the triangles ABC and BCM; note that they have the same height above AM. If you can prove that $AC > CM$, you have won (why?) But $AC > CM$ follows from the fact that $BC = CM$ and the right angle at B.