

A VERY SHORT HISTORY OF CALCULUS

The history of calculus consists of several phases. The first one is the ‘prehistory’, which goes up to the discovery of the fundamental theorem of calculus. It includes the contributions of Eudoxus and Archimedes on exhaustion as well as research by Fermat and his contemporaries (like Cavalieri), who – in modern terms – computed the area beneath the graph of functions like x^r for rational values of $r \neq -1$.

Calculus as we know it was invented (Phase II) by Leibniz; Newton’s theory of fluxions is equivalent to it, and in fact Newton came up with the theory first (using results of his teacher Barrow), but Leibniz published them first. The dispute over priority was fought out between mathematicians on the continent (later on including the Bernoullis) and the British mathematicians.

Essentially all the techniques covered in the first courses on calculus (and many more, like the calculus of variations and differential equations) were discovered shortly after Newton’s and Leibniz’s publications, mainly by Jakob and Johann Bernoulli (two brothers) and Euler (there were, of course, a lot of other mathematicians involved: L’Hospital, Taylor, ...).

As an example of the analytic powers of Johann Bernoulli, let us consider his evaluation of $\int_0^1 x^x dx$. First he developed

$$x^x = e^{x \ln x} = 1 + x \ln x + \frac{x^2(\ln x)^2}{2!} + \frac{x^3(\ln x)^3}{3!} + \dots$$

into a power series, then exchanged the two limits (the power series is a limit, and the integral is the limit of Riemann sums), integrated the individual terms (integration by parts), used the fact that $\lim_{x \rightarrow 0} x^n(\ln x)^n = 0$ and finally came up with the answer

$$\int_0^1 x^x dx = 1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \dots$$

Exchanging limits, transforming power series (even those converging only conditionally) and similar techniques were used without justification.

The foundations of the calculus were shaky, to say the least: neither Newton’s fluxions nor Leibniz’s infinitely small quantities could be given a precise definition. Bishop Berkeley published a famous manuscript in which he pointed out these shortcomings and claimed that calculus was as much based on faith as was theology.

The mathematicians were aware of these problems, but still were mainly concerned with inventing methods for applying calculus: Lagrange, Fourier, Laplace, Gauss, Stokes, Green, Jacobi refined and extended analysis and applied it to problems in celestial mechanics and physics.

Making analysis rigorous (Phase III) was a long process that started in the 19th century, the main contributors being Cauchy (the precise notion of limit), Abel, Dirichlet, Riemann, Weierstrass ($\varepsilon - \delta$), Dedekind (construction of the reals using Dedekind cuts), and Bolzano.

Phase IV involved mathematicians such as Cantor (set theory; construction of the reals with Cauchy sequences), Baire, and Lebesgue: analysis became abstract and used set theory in an essential way.