

# CHINESE, INDIAN, AND ARABIC MATHEMATICS

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## 1. CHINESE MATHEMATICS

One of the earliest mathematicians of China was Liu Hui (ca. 260 AD). His tools were similar to that of Greek mathematicians, and he even proved theorems; in particular he used similar triangles to solve problems in surveying (e.g. finding the distance between two islands): this is reminiscent of Thales' use of similar triangles to measure the height of pyramids, or to the method Eratosthenes used for determining the circumference of the earth. He estimated  $\pi$  by approximating the circle with regular  $n$ -gons for  $n = 92$  and  $184$  and knew the principle of exhaustion for circles; he also worked on the volume of the sphere (apparently, the works of Archimedes were not known in China).

Zu Chongzhi (429-500) computed  $\pi$  to seven digits. Around 600 AD, Indian works on mathematics were translated into Chinese.

Wang Xiaotong (ca. 625) showed how to compute roots of cubic equations numerically. Qin Jiushao (1202 – 1261) treated linear systems of congruences (Chinese remainder theorem) and discussed the Euclidean algorithm for computing greatest common divisors.

Chinese mathematics had a 'silver' period from 300–700, and a 'golden' period during the 13th century (the Chinese version of Pascal's triangle is from this period). Western mathematics was introduced in the 17th century.

## 2. INDIAN MATHEMATICS

Aryabatha (476–550) introduced and tabulated the sine function, and worked out the solution of linear diophantine equations like  $ax + by = c$ , where  $a, b, c$  are integers. Although Aryabatha used letter to denote numbers (like the Greeks), he might also have known the decimal system. He also claimed that the circumference of the earth was about 5000 Yojanas, one Yojana being 7.2 km.

Brahmagupta (598–670) already worked with 0 as a number, as well as with negative numbers (representing debt). He also studied diophantine equations of degree 1 ( $ax + c = by$ ) and 2 ( $ax^2 + c = y^2$ ); in particular he solved the 'Pell equation'  $61x^2 + 1 = y^2$ , whose smallest positive solution is given by  $(x, y) = (226153980, 1766319049)$ . This solution was of course not discovered by trial and error: the Indians developed an algorithm related to our continued fraction algorithm. The first European mathematician who could solve  $61x^2 + 1 = y^2$  was Fermat, who used this problem in one of his challenges to English mathematicians.

Bhaskaracharya (1114–1185) also studied linear diophantine equations (some with three variables, such as  $ax + by + cz = d$ ) as well as Pell equations, and he knew that  $x^2 = 9$  had two solutions, which means he fully accepted negative numbers.

## 3. ARABIC MATHEMATICS

The Arabs preserved, studied, and extended Greek mathematics, in particular Euclidean geometry (the parallel axiom, for example, was studied a lot) and number theory; amicable numbers were another hot topic, and the Arabic mathematicians were the first to claim that  $x^3 + y^3 = z^3$  and  $x^4 + y^4 = z^4$  cannot be solved in natural numbers. The Arabs imported the decimal system from India; Fibonacci (ca. 1200) and others learned it from the Arabs and introduced them in Europe.

Thabit Ibn Qurra (826–901) showed that quadratic equations can be solved using geometric methods. In some sense, this idea is similar to that of Descartes, who showed that geometric problems could be translated into algebra. Omar Khayyam (1048–1131) later expanded these ideas by showing that cubic equations  $x^3 + b^2x + a^3 = cx^2$  could be solved geometrically. Of course, the coefficients  $a, b, c$  are *positive* numbers; moreover, each term in the cubic consists of exactly three factors and therefore could be interpreted geometrically as some volume. The geometric language was also used much later by Cardano in his *Ars Magna*.