# W.S. Anglin.

# Mathematics: A Concise History and Philosophy

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It didn't take me more than a few minutes of browsing through W.S. Anglin's *Mathematics: A Concise History and Philosophy* to decide that this is a book I would not recommend to anyone interested in the history of mathematics. Since it is the only book on the history of mathematics available in the bookstore on the Bilkent campus, I decided to write down a few reasons why I can't stand it. Looking back now it seems that there were more reasons than I expected ...

# God

My first and main complaint about Anglin's book is the fact that it is sprinkled with irrelevant references to God (the Christian version; Anglin worked at the Luther College in Regina, Canada) and the Bible. I will now list and comment upon some of them.

p. 1: Anglin starts with the promotion of his faith on page 1, where he claims that Platonism "sees mathematics descending from a divine realm". Actually, Platonism is the belief that mathematical entities are real, and that they exist independently of us and outside of space and time. As Frege put it:

Thus the thought, for example, which we express in the Pythagorean theorem is timelessly true, true independently of whether anyone takes it to be true.

- p. 2: The Rhind Papyrus "was copied by a scribe called Ahmose in 1650 B.C., about the time Joseph was governor of Egypt". What Anglin doesn't tell us is: How many years is that since Adam and Eve were thrown out of Eden?
- p. 6: When talking about the Sumerian mathematical achievements, he writes

Most of these achievements go back as far as 2000 BC — about the time when Abraham's father was living in the Sumerian city of Ur.

Since this is a book about history, does this mean that the biblical story of Abraham is a historical fact? Actually, even theologians debate about whether the city of Ur mentioned in the Bible is the one located in southern Mesopotamia (as Anglin claims) or some other city in the northern part.

Next Anglin claims, in connection with the Babylonian sexagesimal system, that the scale 60 for weights was endorsed by God himself:

Lord Yahweh says this: ... twenty shekels, twenty-five shekels, and fifteen shekels shall be your mina (Ezekiel 45:9-12).

The part that Anglin left out (...) actually reads:

The shekel shall be twenty gerahs.

Obviously Anglin is a bit selective here and quotes only that part that goes well with his claim. In science, this is called forgery.

It is also a complete mystery to me where he got the idea from that Ezekiel wrote this in 573 BC (and not, say, in 572 BC).

In addition, theologians do not agree on what the quoted part actually means: on http://www.bibleinsight.com/menep1.html I found

This verse is commonly thought to be suggesting the mina of the sanctuary was composed of 60 shekels. However, this view fails to explain why the verse breaks the apparent 60 shekels into twenty, twenty-five and fifteen shekels.

There have been a number of theories which have attempted to explain the text and different renderings have been suggested.

p. 8: Here Anglin tries to bring in God via the formula

$$1^2 + 2^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$
:

The Bible tells us that there was once an attempt to build a ziggurat 'with its top reaching heaven' (Genesis 11:4). Perhaps the promoters of the Tower of Babel mistakenly believed that the infinite series  $1^2 + 2^2 + \ldots + n^2$  converges.

Is this an attempt at humor? Or does he actually believe in a literal interpretation of the Bible? And what about the implication that Babylonians had any notion of convergence?

p. 15: Without the help of parents or teachers, Thales would have done nothing. Nor should we forget God. A theist might claim that if God did not create us and protect us, we would never discover anything.

Not to mention publish books such as Anglin's.

p. 17: Pythagoras studied under the Babylonians, and he may have met the prophet Daniel in Babylon.

Then again, he may have not. What purpose do these speculations serve? And Anglin goes on:

Nor is it impossible that Pythagoras studied in India. . . . Perhaps Pythagoras meth Buddha, another of his contemporaries.

p. 49: Writing about how Justinian closed down the Academy founded by Plato in Athens, he offers the following explanation:

This was because the Academy had failed to accept the new Christian knowledge.

Thus it was not the intolerance of the Christian leaders that was responsible for closing the Academy, but the refusal of the scientists there to accept not the Christian teachings, but the Christian *knowledge*!

And Anglin goes on; on p. 111 we read

Most of the mathematicians at the Academy and the Museum rejected the new truths [sic] of Christ's revelation. This was unfortunate because the split between the old scientific learning and the vibrant new faith weakened the Roman Empire, which was the bulwark of civilisation in the West. If the mathematicians had joined the Christians, the Dark Ages would have been brightened by a dialogue between reason and faith.

Amen.

p. 107: A classic Anglin:

We do not know if Diophantus himself was Christian, but it is not impossible.

This idea is reinforced on page 110:

It is sad that there were so few mathematicians in the early Christian church. Anatolius, and possibly Diophantus, were exceptions.

If you should ask yourself who Anatolius was, the index does not help you since he is not mentioned there. On p. 107 we read that he apparently wrote a book on the Elements of Arithmetic as well as a tract on Egyptian fractions, all of which are lost. Why this makes him a mathematician before the eyes of Anglin I don't know: on p. 158, he writes

In other words, Galileo is like Hypatia. Neither did much for mathematics, . . .

How much Hypatia did for mathematics is not known, since her writings are all lost. So what does Anatolius have that Hypatia does not?

p. 162/163: For reasons unexplained, Anglin gives Descartes "proof of the existence of  $\operatorname{God}$ ":

recurring to the examination of the idea of a Perfect Being, I found that the existence of the Being was comprised in the idea in the same way that the equality of its three angles to two right angles is comprised in the idea of a triangle, or as in the idea of a sphere, the equidistance of all points on its surface from the centre, or even still more clearly; and that consequently it is at least as certain that God, who is this Perfect Being, is, or exists, as any demonstration of geometry can be.

What does Anglin think of Descartes' 'proof'?

Unfortunately for Descartes, there are triangles – in hyperbolic geometry – whose three angles do not add up to two right angles. We need to repair Descartes' argument by adding the word 'Euclidean' before the word 'triangle'.

Proof repaired?

- p. 213: Here Anglin has a field day. Concerning the well ordering principle, "Cantor's faith in God guided him in the right direction", as "Cantor adopted it because he believed there is a God who can arrange the elements of any set so that they are well-ordered". Others were not so lucky: "Some of the early workers in set theory, such as atheist Bertrand Russell (1872–1970), originally thought that no such restriction was necessary".
- p. 225: Here we find the statement that 'Ramanujan sometimes credited his discoveries to providence', and this is followed by his quote that 'An equation for me has no meaning unless it expresses a thought of God.'

Actually Ramanujan credited some of his discoveries to the goddess Namagiri, and not to 'God' as the presentation of Anglin suggests.

# Historical Problems

#### Dodecahedron

On p. 18 we read:

The Pythagoreans discovered the dodecahedron, .... This accomplishment was unsurpassed until J. Kepler (1571–1630) discovered the lesser and greater stellated dodecahedra.

Why did Kepler's discovery 'surpass' that of the Pythagoreans?

# Perfect Numbers

On p. 23, Anglin discusses the fact that  $2^{m-1}(2^m - 1)$  is a perfect number if  $2^m - 1$  is prime. This can be found in Euclid, but

the proof is probably due to the Pythagorean Archytas.

Then he gives "Archytas' proof" without mentioning that it is the one found in Euclid, and even writes

It should be noted that, although Archytas attempted to give a fully rigorous proof of unique factorization for numbers of the form  $2^{m-1}(2^m-1)$ , he failed to do so.

It takes a while until one realizes that he is talking about Euclid's proof here. How the attempted proof goes or what the gap in the proof is, Anglin does not say; I find nothing wrong with it (I have modernized the notation, and did not restrict to  $m \leq 3$  as Euclid had to due to his geometric language):

Assume that P is a divisor of DE, where  $D=2^{m-1}$  and where  $E=2^m-1$  is prime; assume also that P is not in the list  $1, 2, \ldots, 2^{m-1}, E, 2E, \ldots, DE$ . Then PQ=DE for some Q. In IX.13 he has proved that the only divisors of a prime power are powers of this prime; thus the only divisors of D are D, D, D, D, D, we find that D does not divide D. Since D:P=Q:E, we find that D does not divide D. But D is prime and D divides D, therefore D must divide D. Since D:D is a power of D, this shows that D divides D, and by IX.13 we find that D is a power of D, say D with D is a power of D. But then D is a power of D, which is on the list of divisors, contradicting the assumption.

 $\label{lem:http://aleph0.clarku.edu/} $$ $$ \frac{dj}{dj} elements/bookIX/propIX36.html for Euclid's proof. $$$ 

#### **Pythagoreans**

After talking about the Pythagoreans' approximation of  $\sqrt{2}$ , he writes (p. 36)

in essence, it works as follows

and then goes on to talk about reals, some version of the Berlekamp algorithm for computing a Bezout representation of the gcd, and then seems to claim that this was how the Pythagoreans did it:

For example, suppose a Pythagorean wanted to find an integer solution to 17x - 19y = 320. He would reason in a way we would describe as follows:

It is strange then, that no other historian of mathematics is aware of the fact that the Pythagoreans solved linear diophantine equations.

After having ascribed the modern version to the Pythagoreans, he draws the conclusion (p. 38) that

The Pythagoreans had insights that took over 2000 years to comprehend.

#### Wantzel

On p. 50, the biography of P. Wantzel is reduced to the following:

In 1837, a French opium addict, Pierre Wantzel (1814 - 1848) ...

This is probably lifted from

 $\label{lem:http://www-gap.dcs.st-and.ac.uk/} $$ \text{http://www-gap.dcs.st-and.ac.uk/} $$ \text{where we find} $$$ 

According to Saint-Venant ... his death was the result of overwork. Saint-Venant wrote: "... one could reproach him for having been too rebellious against those counselling prudence. He usually worked during the evening, not going to bed until late in the night, then reading, and got but a few hours of agitated sleep, alternatively abusing coffee and opium, taking his meals, until his marriage, at odd and irregular hours."

An opium addict? A forgery. Just in order to make sure the reader gets his point, Anglin repeats it on p. 72,

... but in 1837, a French opium addict, Pierre Wantzel, ...

and once more on p. 204:

Wantzel died young on account of drinking too much coffee and smoking too much opium.

Finally, on p. 78 he seems to claim that the proof that a regular p-gon is constructible with ruler and compass is due to Wantzel (drawing on the work of Gauss). Gauss is credited for the construction of the 17-gon on p. 75. In an exercise on p. 79 it is claimed that the construction of the 771-gon (771 = 3.257) can be done using Wantzel's result; but this is definitely a result due to Gauss.

#### Plato

I like this (p. 57):

About 380 BC, Plato found the Academy.

I guess that's why they call it Platonism.

#### Greeks and Descartes

On p. 77, Anglin gives the Greeks' construction of points and lines in the plane. In particular, he claims they could "multiply two segments" an then describes Descartes' construction from the 1630s. For the Greeks, of course, the product of two line segments would have been not another line (this was Descartes' great invention) but a rectangle!

#### Archimedes

On the death of Archimedes (p. 96/97):

There are various accounts of why this happened.

This is in fact true. But what is the point of inventing yet another one?

Perhaps it was simply because the soldier had watched his best friend being killed by one of Archimedes's machines.

Actually the point is to have an excuse for invoking the Bible:

Those who take the sword die by the sword (Matthew 26:52).

Archimedes' proof that the area of a circle with radius r is  $\pi r^2$  is described in detail on p. 97/98; it is claimed that he proved the following:

A regular  $2^n$ -gon inscribed in a circle takes up more than  $1 - \frac{1}{2^{n-1}}$  of its area. A regular  $2^n$ -gon circumscribed about a circle has an area less than  $1 + \frac{1}{2^{n-2}}$  times that of the circle.

This is pure fiction.

#### Hypatia

On p. 110, Anglin tells the story of Hypatia:

According to Socrates Scholasticus (380-450 AD), in Chapter 15 of Book VII of his *History of the Church*, Hypatia was murdered by a mob of 'Christians', led by one 'Peter'. This tragedy is sometimes blamed on the Christian bishop, Cyril, but there is no evidence to support this accusation. Cyril was a zealous leader, but we have no reason to think he 'incited' the crowd to make a physical attack on the pagan mathematician. Indeed, we have no reason to think that

the murder had anything to do with religion and science. For all we know, the mob killed Hypatia simply because they were poor and unemployed, while Hypatia had a permanent well-paid job.

For all he knows. What an argument. A sample assignment on p. 231 contains the problem

Discuss possible reasons for the death of Hypatia.

Why does Anglin think that speculation in absence of facts has something to do with history?

#### Al-Khwarizmi

Anglin does not like him, to say the least:

Al-Khwarizmi's 'Algebra' contains nothing that was not known to the ancient Greeks. There are few proofs, and one of them is woefully inadequate. This is al-Khwarizmi's 'proof' of the theorem of Pythagoras, which only works if the right triangle is isosceles!

Al-Khwarizmi gives three approximations for  $\pi$ . None of them is supported by any reasoning, and Al-Khwarizmi does not seem to care which one is used. Al-Khwarizmi was a transmitter of ancient Greek knowledge, not an original mathematician.

I wonder what the verdict would have been had Al-Khwarizmi had the same religion as Anatolius. The essay question of this chapter is

Who has a better right to the title 'Father of Algebra', and why: Diophantus or al-Khwarizmi?

This question seems to be lifted from

http://www-gap.dcs.st-and.ac.uk/ history/Mathematicians/Al-Khwarizmi.html, where we read

Al-Khwarizmi's algebra is regarded as the foundation and cornerstone of the sciences. In a sense, al-Khwarizmi is more entitled to be called "the father of algebra" than Diophantus because al-Khwarizmi is the first to teach algebra in an elementary form and for its own sake, Diophantus is primarily concerned with the theory of numbers.

Apparently, Anglin's presentation has the goal to convince the reader that the opposite is true (which I would support were it not for the fact thay Anglin works with rhetoric instead of arguments). How the reader is supposed to answer such a question when the book doesn't give any details about either mathematician's work, I have no idea.

#### Galilei

Apparently, Galilei is Anglin's personal enemy. I have no idea why he insists on addressing him by his first name. Anyway, on p. 157 we read

[Galileo] did not make any original contributions to mathematics. . . . So if he did non contribute to mathematics, what is Galileo doing here?

And he answers his own question as follows:

Galileo is sometimes included in histories of mathematics because the anti-Catholic historian wants the chance to tell everyone how badly the Catholic Church treated Galileo.

He also hastens to remark that "From Einstein's point of view, then, it seems silly that Galileo Galilei (1564–1642) and the Inquisition fought over whether the earth goes around the sun or vice versa".

As if this were not low enough, Anglin goes on with his Essay Questions:

- 1. Psalm 104 praises God 'who laid the foundations of the earth, so that it should not be moved forever'. Write a short essay showing that this verse can be interpreted in a way that respects both the truth of God's relevation and the truth of Science.
- 2. Comment on the following. Socrates and Jesus were willing to die for what they believed was right, but Galileo recanted because he was a coward.

Although Anglin would like to banish Galilei (who could be called the father of mathematical physics) from books on the history of mathematics, he devotes a page to Kepler's astronomy (p. 158/159) and another page to Newton's law of gravity (p. 177/178).

#### Fermat's Last Theorem

In connection with Fermat's Last Theorem, Anglin writes on p. 165:

In 1823, Legendre disposed of the case with n=5, and, in 1832, Dirichlet handled the case with n=7.

Actually the case n=5 is due to Legendre and Dirichlet, whereas the case n=7 was handled by Lamé. Dirichlet covered the case n=14 in 1832.

### Pascal and Bell

Writing about Pascal, Anglin complains bitterly that

because of his religious priorities, historians such as E.T. Bell have branded him a madman.

Whether Bell should be called a historian, I don't know; it should be remarked, however, that Bell didn't call Pascal a madman but, according to Anglin, a 'religious neurotic'.

Anglin then he gives Pascal's wager in detail, but doesn't say a word about its flaws. In a paragraph titled 'The Real Madman', Anglin attributes several conclusions to Bell, starting with the following:

(1) a person who lacks reason is an expert at mathematical reasoning:

What Bell was saying is that Pascal, a "highly gifted mathematician", wasted a lot of time by thinking about "meaningless mysticism and platitudinous observations on the misery and dignity of man". As for (1), may I recommend the biography of Nash?

Finally, Anglin comes forward with the following sentence:

These conclusions are insane, and one might well raise some questions about Bell's mental state.

#### Barrow and Calculus

This here is a gem from p. 176:

Isaac Barrow [...] was the first mathematician to realise that

$$\int_{a}^{b} f'(x)dx = f(b) - f(a).$$

This is the fundamental theorem of calculus.

Makes you wonder what Newton and Leibniz were fighting over.

# **Great Number Theorists**

Anglin's list of 19th-century number theorists (p. 209) consists of Gauss and Cauchy, Dirichlet, Sylvester (for his next to trivial proof that every fraction can be written as a sum of Egyptian fractions), Hadamard and de la Vallée-Poussin, and Lucas, who studied the diophantine equation  $6y^2 = x(x+1)(2x+1)$  for which a "simple elementary proof [...] was first given in 1990 by W.S. Anglin". The review in Mathematical Reviews mentions that simple elementary proofs were given before by Ma (1985) and Xu & Cao (1985).

Anglin also is under the impression that the proof of Fermat's conjecture that every number is the sum of three triangular numbers, four squares, five pentagonal numbers etc. is "one of the major achievements of nineteenth-century number theory". Quadratic reciprocity is not mentioned at all, neither is the fundamental work of Eisenstein, Kummer, Dedekind, Weber, or Hilbert.

# Alexandria's Library

Let us start with the following claim about the library in Alexandria on p. 81:

This university, called the 'Museum', soon had a library with more than 600,000 papyrus rolls. This library was destroyed by the Arabs in 641 A.D.

On p. 110 he writes

In 641 A.D., Alexandria fell to the Arabs, who burned the famous library.

What he fails to mention is that most of the 600,000 papyrus rolls he is talking about perished long before 641. Moreover, the fact that the Arabs burned destroyed the library is not a fact at all.

The origin of this legend is the following story:

As for the books you mention, here is my reply. If their content is in accordance with the book of Allah, we may do without them, for in that case the book of Allah more than suffices. If, on the other hand, they contain matter not in accordance with the book of Allah, there can be no need to preserve these. Proceed, then, and destroy them.

Actually, this story (along with others who blame the Christians in 391 for the destruction) is just a legend. Let me quote a letter to the editor of the New York Times:

#### • http://www.nybooks.com/articles/3517

From Professor Hugh Lloyd-Jones's review of Luciano Canfora's book on the library of Alexandria [NYR , June 14], one learns, with astonishment, that the author, and perhaps even to some degree the reviewer, are still disposed to lend credence to the story of how the great library of Alexandria was destroyed by the Arabs after their conquest of the city in 641 AD, by order of the Caliph 'Umar.

This story first became known to Western scholarship in 1663, when Edward Pococke, the Laudian Professor of Arabic at Oxford, published an edition of the Arabic text, with Latin translation, of part of the History of the Dynasties of the Syrian-Christian author Barhebraeus, otherwise known as Ibn al-'Ibri. According to this story, 'Amr ibn al-'As, the commander of the Arab conquerors, was inclined to accept the pleas of John the Grammarian and spare the library, but the Caliph decreed otherwise: "If these writings of the Greeks agree with the book of God, they are useless and need not be preserved; if they disagree, they are pernicious and ought to be destroyed." The books in the library, the story continues, were accordingly distributed among the four thousand bathhouses of the city, and used to heat the furnaces, which they kept going for almost six months.

As early as 1713, Father Eusèbe Renaudot, the distinguished French Orientalist, cast doubt on this story, remarking, in his History of the Patriarchs of Alexandria published in that year, that it "had something untrustworthy about it." Edward Gibbon, never one to miss a good story, relates it with gusto, and then proceeds: "For my own part, I am strongly tempted to deny both the fact and the consequences." To explain this denial, Gibbon gives the two principal arguments against authenticity: that the story first appears some six hundred years after the action which it purports to describe, and that such action is in any case contrary to what we know of the teachings and practice of the Muslims.

Since then, a succession of other Western scholars have analyzed and demolished the story: Alfred J. Butler in 1902, Victor Chauvin in 1911, Paul Casanova and Eugenio Griffini, independently, in 1923. Some have attacked the internal improbabilities of the story. A large proportion of books of that time would have been written on vellum, which does not burn. To keep that many bathhouse furnaces going for that length of time, a library of at least 14 million books would have been required. John the Grammarian who, according to the Barhebraeus story, pleaded with 'Amr for his library, is believed to have lived and died in the previous century. There is good evidence that the library itself was destroyed long before the Arabs arrived in Egypt. The 14th century historian Ibn Khaldun tells an almost identical story concerning the destruction of a library in Persia, also by order of the Caliph 'Umar, thus demonstrating its folkloric character. By far the strongest argument against the story, however, is the slight and late evidence on which it rests. Barhebraeus, the principal source used by Western historians, lived from 1226 to 1289. He had only two predecessors, from one of whom he simply copied the story and both preceded him by no more than a few decades. The earliest source is a Baghdadi physician called 'Abd al-Latif, who was in Egypt in 1203, and in a brief account of his journey refers in passing to "the library which 'Amr ibn al-'As burnt with the permisison of 'Umar." An Egyptian scholar, Ibn al-Qifti, wrote a history of learned men in about 1227, and includes a biography of John the Grammarian in the course of which he tells the story on which the legend is based. His narrative ends: "I was told the number of bathhouses that existed at that time, but I have forgotten it. It is said that they were heated for six months. Listen to this story and wonder!" Barhebraeus merely followed the text of Ibn al-Qifti, omitting his final observation on the number of baths. This number is provided by other Arabic sources, in quite different contexts.

To accept the story of the Arab destruction of the library of Alexandria, one must explain how it is that so dramatic an event was unmentioned and unnoticed not only in the rich historical literature of medieval Islam, but even in the literatures of the Coptic and other Christian churches, of the Byzantines, of the Jews, or anyone else who might have thought the destruction of a great library worthy of comment. That the story still survives, and is repeated, despite all these objections, is testimony to the enduring power of a myth.

Myths come into existence to answer a question or to serve a purpose, and one may wonder what purpose was served by this myth. An answer sometimes given, and certainly in accord with a currently popular school of epistemology, would see the story as anti-Islamic propaganda, designed by hostile elements to blacken the good name of Islam by showing the revered Caliph 'Umar as a destroyer of libraries. But this explanation is as absurd as the myth itself. The original sources of the story are Muslim, the only exception being Barhebraeus, who copied it from a Muslim author. Not the creation, but the demolition of the myth was the achievement of European scholarship, which from the 18th century to the present day has rejected the story as false and absurd, and thus exonerated the Caliph 'Umar and the early Muslims from this libel.

But if the myth was created and disseminated by Muslims and not by their enemies, what could possibly have been their motive? The answer is almost certainly provided in a comment of Paul Casanova. Since the earliest occurrence of the story is in an allusion at the beginning of the 13th century, it must have become current in the late 12th century, that is to say, in the time of the great Muslim hero Saladin, famous not only for his victories over the Crusaders, but also – and in a Muslim context perhaps more importantly – for having extinguished the heretical Fatimid caliphate in Cairo, which, with its Isma'ili doctrines, had for centuries threatened the unity of Islam. 'Abd al-Latif was an admirer of Saladin, whom he went to visit in Jerusalem. Ibn al-Qifti's father was a follower of Saladin, who appointed him Qadi in the newly conquered city.

One of Saladin's first tasks after the restoration of Sunnism in Cairo was to break up the Fatimid collections and treasures and sell their contents at public auction. These included a very considerable library, presumably full of heretical Isma'ili books. The break-up of a library, even one containing heretical books, might well have evoked disapproval in a civilized, literate society. The myth provided an obvious justification. According to this interpretation, the message of the myth was not that the Caliph 'Umar was a barbarian because he destroyed a library, but that destroying a library could be justified, because the revered Caliph 'Umar had approved of it. Thus once again, as on so many occasions, the early heroes of Islam were mobilized by later Muslim tradition to give posthumous sanction to actions and policies of which they had never heard and which they would probably not have condoned.

It is surely time that the Caliph 'Umar and 'Amr ibn al-'As were finally acquitted of this charge which their admirers and later their detractors conspired to bring against them.

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# Abstraction

Anglin has a solid aversion against abstract concepts. On p. 183 we read

Nonstandard analysis is a complicated and bizarre system. It seems too ugly to be true.

Here are some quotes from the introductions to Chapters 39

On the other hand, much twentieth century mathematics was characterised by a degree of abstraction never seen before. It was not

the Euclidean plane that was studied, but the vector spaces and topological spaces which are abstractions of it. It was not particular groups that were studied so much as the whole 'category' of groups.

#### and Chapter 40:

Much of what went under the name 'number theory' in the twentieth century had little to do with natural numbers. There was an obsession with results concerning abstract structures to prove results concerning abstract structures. ... A few number theorists escaped the obsession with abstraction and produced the meaningful concrete results listed below.

It strikes me as weird that among these number theorists doing something meaningful we find, among others, Tunnell, Ribet, Serre and Wiles. If these weren't 'obsessed' with abstract structures, who does Anglin blame?

Not himself, that much is sure, because he manages to list himself here with a result on angles in Pythagorean triangles.

The essay question of Chapter 40 is

Because they must 'publish or perish', second-rate mathematicians fill the journals with useless abstractions, calling their work 'number theory' when it is merely jejune generalisation. Can you suggest some replacement for the 'publish or perish' system that is currently cluttering our libraries with junk?

Who is Anglin to look down upon the mathematics he doesn't understand? Let me mention (not as a defense for the nonsense put forward by Anglin) that he was not the first to complain about the sometimes merciless abstraction of the mathematics of the 20th century. In a letter to Mordell about a review of Lang's book on diophantine geometry, Siegel writes

Thank you for the copy of your review of Lang's book. When I first saw this book, about a year ago, I was disgusted with the way in which my own contributions to the subject had been disfigured and made unintelligible. My feeling is very well expressed when you mention Rip van Winkle!

The whole style of the author contradicts the sense for simplicity and honesty which we admire in the works of the masters in number theory—Lagrange, Gauss, or on a smaller scale, Hardy, Landau. Just now Lang has published another book on algebraic numbers which, in my opinion, is still worse than the former one. I see a pig broken into a beautiful garden and rooting up all flowers and trees.

Unfortunately there are many "fellow-travelers" who have already disgraced a large part of algebra and function theory; however, until now, number theory had not been touched. These people remind me of the impudent behaviour of the national socialists who sang: "Wir werden weiter marschieren, bis alles in Scherben zerfaellt!"

I am a fraid that mathematics will perish before the end of this century if the present trend for senseless abstraction—as I call it: theory of the empty set—cannot be blocked up. Let us hope that your review may be helpful...

It were exactly these 'senseless' abstractions that allowed number theorists who understood them to prove classical conjectures by Mordell, Fermat, and others. History has proved Siegel wrong. But at least Siegel was a first rate mathematician; Anglin is not.

# Reviews of Anglin's Book

### John N. Crossley

Do you want to know how Euler died, why Hypatia was killed, that Cantor and Cauchy both wrote poems to their wives and who were Christian Mathematicians?

You may find out the answers in this book. On the other hand you may not. For example, Anglin only surmises that Hypatia may have been killed because she had a well paid job.

Thus far it is clear that this book is tendentious and journalistic in style. To give credit where it is due there is a succinct and commendable account of the discovery and publication of the solution of the cubic by Cardano and others in the sixteenth century. (By the way you have to know that the text reference to "Ars Magna" means "The Great Art" in the list of references).

As to the book's substance here is a sample. In Chapter 25, p. 147, "we give the solution to the cubic equation that is essentially that of Ferro, Tartaglia, and Viete." This is amazing. Ferro had one partial solution and as the author says on p. 142, Tartaglia could solve more cases, while Viete used a different method entirely. None of them used cube roots of unity as does the author.

So what Anglin does is in fact show is how you (or he) can solve a cubic. Fair enough, but not if you are claiming as Anglin does in the Preface that this is one of the "many detailed explanations of the important mathematical procedures actually used by famous mathematicians".

The book is replete with exercises and essay questions. My favourite exercise is no. 1 in Exercises 21: "Prove the Trivial Theorem." However it does not take long to find which theorem is really meant, for the chapter only has two pages of text - not the only chapter of this brevity, nor the shortest.

This book is not a deep study, but someone who did all the exercises and essays - there are lots - would have learnt lots of history of mathematics by the end of those. The text and references would not provide a very good starting point and the student would have to work out which reference was relevant since Anglin rarely cites his sources.

For philosophy one would need even more work; philosophy essentially occupies a mere four pages: 206 and 217-219.

The book is "accessible to students who have trouble coping with vast amounts of reading." Indeed, the main text occupies less than 150 pages.

Finally, the last essay question (p. 225) ends: "Can you suggest some replacement for the 'publish or perish' system that is currently cluttering our libraries with junk?" - and, one might add, this book?

#### ibe

This is a well written and useful textbook for an introductory one-semester course in the history and philosophy of mathematics. It contains many detailed explanations of important mathematical procedures actually used by famous mathematicians and gives an opportunity to learn the history and philosophy by way of problem solving. In 40 short chapters, various kinds of interesting mathematical topics are discussed (from ancient to 20th century: unit fractions, pythagorean mathematics, figurative numbers, five regular solids, golden ratio, diophantine equations, Fibonacci numbers, cubic equations, four square theorem, Cantor's set theory, etc.). The reader will find in the book also basic information about great mathematicians. The book includes bibliographical references (26 titles) and an index. In Appendix A, there are sample assignments and tests (15 pp.); answers to selected exercises (11 pp.) are found in Appendix B. The book can be warmly recommended both to secondary and high school mathematics teachers as well as to students and to everyone interested in the history of mathematics.

#### **Mathematical Reviews**

This is a concise introductory textbook in the history and philosophy of mathematics. It is designed for two purposes, viz. (i) to help students of mathematics to acquire a philosophical and cultural background by doing actual mathematical problems from different eras, and (ii) to help them come to a deeper understanding of mathematical culture by means of writing articles. Thus this book aims at giving a concise integrated view of mathematics by approaching the subject through its history, philosophy and culture. The book has four special features, viz. (a) it is short and easily accessible; (b) some chapters deal with detailed explanations of important mathematical procedures; (c) several important philosophical topics are emphasized; (d) it offers a deep penetration into key mathematical and philosophical aspects of the history of mathematics.

The author divides this book into forty chapters. They are as follows: (1) Mathematics for the civil servants, (2) The earliest number theory, (3) The dawn of deductive mathematics, (4) The Pythagoreans, (5) The Pythagoreans and perfection, (6) The Pythagoreans and polyhedra, (7) The Pythagoreans and irrationality, (8) The need for the infinite, (9) Mathematics in Athens before Plato, (10) Plato, (11) Aristotle, (12) In the time of Eudoxus, (13) Ruler and compass constructions, (14) The oldest surviving math book, (15) Euclid's geometry continued, (16) Alexandria and Archimedes, (17) The end of Greek mathematics, (18) Early medieval number theory, (19) Algebra in the early Middle Ages, (20) Geometry in the early Middle Ages, (21) Khayyam and the cubic, (22) The latter Middle Ages, (23) Modern mathematical notation, (24) The secret of the cubic, (25) The secret revealed, (26) A new calculating device, (27) Mathematics and astronomy, (28) The seventeenth century, (29) Pascal, (30) The seventeenth century II, (31) Leibniz, (32) The eighteenth century, (33) Lagrange, (34) Nineteenth century algebra, (35) Nineteenth century analysis, (36) Nineteenth century geometry, (37) Nineteenth century number theory, (38) Cantor, (39) Foundations, (40) Twentieth century number theory.

Biographical notes have been inserted at the end of some of the chapters, partly for the sake of human interest. But the notes also help to trace the transmission of ideas from one mathematician to another. The book is well

written and will help those who look for a deeper understanding of mathematical culture.

Reviewed by Pradip Kumar Majumdar

#### Zentralblatt

Das vorliegende Buch stellt eine in 40 Sequenzen - Vorlesungseinheiten entsprechend – gegliederte Einführung in die Geschichte und die philosophische Gedankenwelt der Mathematik dar. Es führt von den mathematischen Ansätzen der Ägypter und Babylonier zur Mathematik der griechischen Antike, die in 14 Kapiteln sehr ausführlich behandelt wird. Der Autor verdeutlicht an einigen sehr gut gewählten Beispielen, wie schade es ist, dass die moderne Schulmathematik diese Gedankenwelt sträflich vernachlässigt. Die europäische Mathematik des 18. und 19. Jahrhunderts wird im Gegensatz zur breiten Diskussion der antiken Mathematik von nur 7 Kapiteln abgedeckt – es ist klar, dass eine thematische Auswahl unumgänglich ist. Viele der angeschnittenen mathematischen Themen sind auch für geschulte Mathematiker von Interesse. Es ist sehr zu begrüßen, dass der Autor – unter Verwendung der heutigen Bezeichnungen und z.T. auch moderner Methoden – einige der reizvollen Themen ausführlich beschreibt. Dies betrifft nicht nur die von Aristarch und Eratosthenes ersonnenen Vermessungsmethoden des Planetensystems, auch Eulers Versuch eines Beweises, dass jede natürliche Zahl die Summe von vier Quadraten ist, und wie Lagrange die Beweislücke füllen konnte. Oder es wird gezeigt, wie Leibniz bei Reihensummationen oft nur zufällig das Richtige traf.

Was den Anspruch des Buches betrifft, philosophische Aspekte aufzuzeigen, ist dem Autor keine so glückliche Hand beschieden: Zum Beispiel ermöglicht – ganz im Gegensatz zu seiner Ansicht – nicht die Leugnung, sondern gerade die Annahme des Akual-Unendlichen Zenons Paradoxien. Unkorrektheiten und Verfälschungen wie diese ziehen sich durch das ganze Buch und enden bei der Diskussion des Intuitionismus mit einer irrigen Beschreibung der Philosophie Kants.

Ebenso ist die Gewichtung der einzelnen Themen nicht immer glücklich: über die Entwicklung des Dezimalsystems wird zum Beispiel nichts berichtet, dafür wird zweimal auf die (bestenfalls als Kuriosität zu wertende) Pellsche Gleichung  $x^2-410286423274424\cdot y^2=1$  eingegangen. Dass der Autor im Haupttext seinen Namen zweimal einschmuggelt, Poincaré dafür z.B. aber nur einmal erwähnt und über Riemann vor allem die süßliche Geschichte seines Dahinscheidens erzählt, ist bereits ärgerlich. Peinlich berührt schließlich das unterschwellige Bemühen des Autors um den positiven Beitrag der Religion in der Geschichte der Mathematik. Dies beginnt mit einer völlig verfehlten Erklärung des Untergangs der hellenistischen Mathematik zu Beginn der christlichen Ära und endet in der Erklärung, dass Cantor aus seiner religiösen Überzeugung heraus Paradoxien in der Mengenlehre zu vermeiden wusste, während der "Atheist Bertrand Russell" ihnen hilflos ausgeliefert war. Bezeichnenderweise wird laut Index der liebe Gott am häufigsten im Text genannt.

R.J. Taschner (Wien)