

QUIZ 5

MATH 111

(1) Compute the following integrals:

(a) $\int \frac{(\sin \sqrt{x+1})(\cos \sqrt{x+1})}{\sqrt{x+1}} dx$

Substitute $y = \sqrt{x+1}$; then $du = \frac{1}{2\sqrt{x+1}} dx$, hence

$$\int \frac{(\sin \sqrt{x+1})(\cos \sqrt{x+1})}{\sqrt{x+1}} dx = 2 \int (\sin u)(\cos u) du = \sin^2 \sqrt{x+1} + C.$$

(b) $\int x f'(3x^2 + 1) dx$, where f is a differentiable function.

Substitute $u = 3x^2 + 1$; then $du = 6x dx$, hence

$$\int x f'(3x^2 + 1) dx = \frac{1}{6} f'(u) du = \frac{1}{6} f(u) + C = \frac{1}{6} f(3x^2 + 1) + C.$$

(2) Compute the area enclosed by the graphs of the functions $y = x^2$ and $y = x - x^2$.

The function $y = x - x^2 = -x^2 + x$ is a parabola bent downwards: you should know how to graph this! It has zeros at $x = 0$ and $x = 1$.

To find the intersection with the other parabola, put $x^2 = x - x^2$, and you get $x = 0$ and $x = \frac{1}{2}$. Since $x - x^2$ is the top function, the area is

$$\int_0^{1/2} (x - x^2 - x^2) dx = \int_0^{1/2} (x - 2x^2) dx = \left. \frac{1}{2}x^2 - \frac{2}{3}x^3 \right|_0^{1/2} = \frac{1}{8} - \frac{1}{12} = \frac{1}{24}.$$