PROBLEMS

MATH 111

Q. 403-10. The base of a solid is the disk $x^2 + y^2 \leq 1$. The cross sections by planes perpendicular to the y-axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disc. Compute the volume of the solid.

A. Obviously we're talking about cross sections here. The volume is $\int_{-1}^{1} A\ dy$, where A is the area of the triangle as a function of y.

The length of the leg is $2x$, where $x^2 + y^2 = 1$, hence $x = \sqrt{1 - y^2}$. The area of the right triangle is $A(y) = \frac{1}{2}(2x)^2 = 2(1 - y^2)$. Thus

$$V = 2\int_{-1}^{1} (1 - y^2)\ dy = \left(2y - \frac{2}{3}y^3\right)\Big|_{-1}^{1} = \frac{8}{3}.$$