PROBLEMS SECTION 5.3

MATH 111

(1) 420-9: Find the length of the curve defined by $x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$ for $-\pi/4 \le y \le \pi/4$.
(2) 420-12: Find the length of the parametrized curve $x = \cos t$, $y = t + \sin t$, $0 \le t \le \pi$.
(3) 420-13: Find the length of the parametrized curve $x = t^3$, $y = 3t^2/2$, $0 \le t \le \sqrt{3}$.

(1) 420-9: Find the length of the curve defined by $x = \int_0^y \sqrt{\sec^4 t - 1}\ dt$ for $-\pi/4 \le y \le \pi/4$.

Here x is a function of y, so the length is $L = \int_{-\pi/4}^{\pi/4} \sqrt{1 + (dx/dy)^2}\ dy$.

By the fundamental theorem of calculus we have $\frac{dx}{dy} = \sqrt{\sec^4 y - 1}$, hence we get

$$
\begin{aligned}
L &= \int_{-\pi/4}^{\pi/4} \sqrt{1 + (dx/dy)^2}\ dy \ = \int_{-\pi/4}^{\pi/4} \sqrt{\sec^4 y}\ dy \\
&= \int_{-\pi/4}^{\pi/4} |\sec^2 y|\ dy \ = \int_{-\pi/4}^{\pi/4} \sec^2 y\ dy \ = \tan y|_{-\pi/4}^{\pi/4} = 2.
\end{aligned}
$$

Here we could omit the absolute value signs since $\sec^2 y \ge 0$.

(2) 420-12: Find the length of the parametrized curve $x = \cos t$, $y = t + \sin t$, $0 \le t \le \pi$.

We find $L = \int_0^\pi \sqrt{\sin^2 t + (1 + \cos t)^2} dt = \int_0^\pi \sqrt{2 + 2\cos t}$. Substitute $u = \cos t$ to compute the integral.

(3) 420-13: Find the length of the parametrized curve $x = t^3$, $y = 3t^2/2$, $0 \le t \le \sqrt{3}$.

We find $L = \int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + (3t)^2}\ dt = 3\int_0^{\sqrt{3}} t\sqrt{1 + t^2} dt$. Using $u = t^2 + 1$ we find $du = 2tdt$, hence $L = \frac{3}{2}\int_1^4 \sqrt{u} du = u^{3/2}|_1^4 = 7$.