

- (1) Let $y = f(x)$ be a differentiable function with $f(0) = 0$ such that its length between $x = 0$ and $x = a$ is given by $L = \sqrt{2}a$ for all $a > 0$. Determine all functions f with this property.
- (2) A 1 m wire is used to make a square and an equilateral triangle. Find the minimal and maximal possible areas.
- (3) Find $f(4)$ if $\int_0^{x^2} f(x) dx = x \cos \pi x$.
- (4) The graph $y = \sqrt{x}$ and the lines $x = \frac{1}{2}$, $y = 0$, and $y = 1$ cut out two regions, which are rotated about $x = \frac{1}{2}$ to generate a solid. Compute its volume.
- (5) Compute $\int \sqrt{\frac{x-1}{x^5}} dx$.