CALCULUS I

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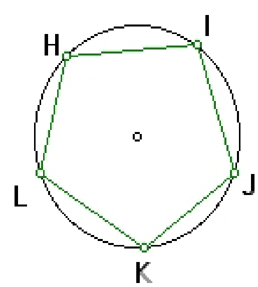
The following problems go back to the Greeks (Euclid and Archimedes), who solved them without calculus.

(1) Consider a circle with radius 1, and an inscribed regular n-gon (the picture shows a regular pentagon). Show that the area of the polygon is $A_n = \frac{1}{2}nr^2 \sin\frac{2\pi}{n}$. Compute $\lim_{n\to\infty} A_n$. Does the result agree with your intuition?



(2) Consider a parabola described by $y = p\sqrt{x}$. The tangent of the parabola in the point $P = (r, s)$ is, according to Euclid and Appolonius, the line RP, where $R = (-r, 0)$. Prove this.

(3) Consider the parabola $y = ax^2$ for some $a > 0$. Pick two different points $P = (p, ap^2)$ and $Q = q, aq^2)$ on this parabola. Let M be the midpoint of the chord PQ. Let R be the point on the parabola with the same x-coordinate as M. Show that the tangent at M is parallel to PQ.

The following problem is not of ancient origin: Determine the asymptote of the function $f(x) = \sqrt{x^4 + x^3 + x^2} - x^2$.