I will not give any diagrams here (way too much work). You, on the other hand, should definitely draw a diagram first.

Another tip: on your homework, always use a complete sentence to state the final result.

1. PROBLEMS

293-9 Choose a linearization with center not $x = a$ but at a nearby value at which the function and its derivative are easy to evaluate. State the linearization and the center.

329-25 Evaluate $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt$.

329-43 The velocity of a particle moving back and forth on a line is $v = \frac{ds}{dt} = 6 \sin 2t \frac{m}{sec}$ for all t. If $s = 0$ when $t = 0$, find the value of s when $t = \pi/2$ sec.

339-17 Estimate the average value of $f(x) = x^3$ on the interval $[0, 2]$ by partitioning this interval into four subintervals of equal length and evaluating f at the subinterval midpoints.

Solutions and Comments

293-9: The task is to linearize the function $f(x) = 2x^2 + 4x - 3$ at a point that is close to $a = -0.9$ and easy to evaluate. This suggests linearizing at $x = -1$.

Linearizing means replacing the function with its tangent in the vicinity of some point. We will therefore need the derivative $f'(x) = 4x+4$. The tangent at $x = -1$ is given by $y = f'(-1)(x+1) + f(-1) = -5$. This is a line parallel to the x-axis.

We get an approximation for $f(-0.9)$ by plugging $x = -0.9$ into the equation of the tangent: thus $f(0.9) \approx -5$. Actually, $f(0.9) = -4.98$.

The linearization of f with center $x = -1$ is $y = -5$.

329-25: I will give three possible attempts. With a little practice you will be able to see in advance which of them will be the most promising.

(1) $u = 2t + 1$. Then $du = 2dt$, hence $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} = \frac{1}{2} \int \frac{\sin u}{\cos^2 u} du$. This looks better than the original integral, but we did not get rid of the sines and cosines.

(2) $u = \sin(2t+1)$. Then $du = 2\cos(2t+1)dt$, hence $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} = 2 \int \frac{udu}{\cos(2t+1)^3}$. But here we still have to replace $\cos(2t+1)$ by a function of u. This will involve $\cos(2t+1) = \sqrt{1 - \sin^2(2t+1)}$, which will not simplify the expression at all.

(3) $u = \cos(2t+1)$. This is a good choice because the derivative of the cosine occurs in the numerator of the integrand. And in fact we get $du = -2\sin(2t+1)dt$, hence

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt = -\frac{1}{2} \int \frac{du}{u^2} = \frac{1}{4u} = \frac{1}{4\cos(2t+1)}.$$

329-43: We know that velocity is the derivative of the function s giving the position. We know $\frac{ds}{dt} = 2\sin 2t$; we need to fint the functions having this as their derivative, that is, we want $s(t) = \int 2\sin 2t = -\cos 2t + C$ for some constant C. We can compute C from the initial value $s(0) = 0$, since this gives $0 = s(0) = -\cos 0 + C = C - 1$. Thus $C = 1$ and $s(t) = 1 - \cos 2t$. In particular, $s(\pi/2) = 1 - \cos\pi = 2$: the value of s after $\pi/2$ seconds is 2 m.

339-17: Let us first use rectangles below the graph. Their area is

$$a = 0.5 \cdot f(0.5) + 0.5 \cdot f(1) + 0.5f(1.5) = 0.5(0.125 + 1 + 3.375) = 2.25.$$

Using rectangles covering the graph we find the following upper bound for the area below the graph of f:

$$A = 0.5 \cdot f(0.5) + 0.5 \cdot f(1) + 0.5f(1.5) + 0.5f(2) = 6.25.$$

As you can see, lower and upper bound differ considerably. This is to be expected if the function grows sharply (our f grows from 0 to 1 on $[0, 1]$ and from 1 to 8 on $[0, 2]$. To get better results, we would need to use finer partitions.

The mean of upper and lower bound, namely $\frac{2.25+6.25}{2} = 4.25$ might be a good estimate of the area.

The average value is by definition the area below the graph divided by the length of the interval, that is, $\approx \frac{4.25}{2} = 2.125$.

Now let us do what the problem wants us to do: we divide $[0, 2]$ into four intervals of length 0.5, but this time we use rectangles whose height is given by evaluating the function at the midpoints of the subintervals, that is, at $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$ and $\frac{7}{4}$. We find that the area below the graph is $\approx \frac{1}{2}[f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})] = \frac{1}{2}\frac{1}{64}[1 + 27 + 125 + 343] = 3.875$.

The average value of f on $[0, 2]$ is therefore approximately $\frac{3.875}{2} = 1.9375$.

The exact value of the area, by the way, is $\int_0^2 x^3 dx = \frac{1}{4}x^4|_0^2 = 4$, and the average value of the function equals $\frac{4}{2} = 2$.