Math 535 – Topology of Algebraic Varieties, Fall 2010
Midterm I (due on October 27, 2010)

Name: ______________________________
I.D.: ______________________________

Problem 1. Let X, Y be two compact complex algebraic curves and $f\colon X \to Y$ an analytic map. Show that $g(X) \geqslant g(Y)$. What can you say about X, Y, and f if $g(X) = g(Y)$?

Does the inequality $g(X) \geqslant g(Y)$ still hold if f is not assumed analytic, just continuous? What exactly do you use in the proof?

Problem 2. As shown in class, the space of plane cubics has dimension 9. Hence, for any set of 9 pairwise distinct points, there is a cubic passing through all these points. (Explain in details.) How many such cubics are there? Does the answer depend on the points? (This problem illustrates the classical notion of *superabundance* of a linear system.)

Problem 3. Show that a plane sextic cannot have the set of singularities $9A_2+A_1$ or $10A_2$. Show that a plane quintic cannot have the set of singularities $5A_2 + A_1$ or $6A_2$.

Problem 4. Compute the invariant δ_c for all singularities that an irreducible plane quartic may have. List all (sets of singularities of) irreducible quartics that may be *self-dual*, i.e., be projectively equivalent to their dual curves.

Problem 5. Show that the invariant $\delta := \frac{1}{2}(\mu + r - 1)$ is semicontinuous in the sense that, if $f_\lambda(x, y)$ is an analytic family of germs with isolated singular points P_λ at the origin $(x, y) = (0, 0)$ (λ being a parameter), then $\delta(P_\lambda) \geqslant \delta(P_0)$ for all λ sufficiently close to 0. (I do not want technical details, but try to explain the phenomenon. *Hint*: what is the topological relation between Milnor fibers of P_λ and P_0? A version of Problem 1 may help at some point.)