Date: November 7, 2002
Time: 1:40–3:30 pm
NAME: ___________________________
STUDENT ID: ______________________

MATH 220-05 MIDTERM I

IMPORTANT
1. This exam consists of 5 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

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1. Solve the system and find a basis for the solution space of the corresponding homogeneous system:

\[
\begin{align*}
2x_1 & - x_2 + x_3 + 2x_4 + 3x_5 = 2 \\
6x_1 & - 3x_2 + 2x_3 + 4x_4 + 5x_5 = 3 \\
6x_1 & - 3x_2 + 4x_3 + 8x_4 + 13x_5 = 9 \\
4x_1 & - 2x_2 + x_3 + x_4 + 2x_5 = 1
\end{align*}
\]
2. Let $A$ and $B$ be, respectively, an $(m \times q)$-matrix and a $(q \times n)$-matrix. Prove that

$$\text{rank}(AB) \leq \min\{\text{rank } A, \text{rank } B\}.$$
3. If exists, find $A^{-1}$ for

$$A = \begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & 3 & 1 & 2 \\
1 & 2 & -1 & 1 \\
5 & 9 & 2 & 6
\end{bmatrix}.$$
4. Find the dimension and a basis for \( \text{Span} \, S \), where

\[
S = \left\{ \begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -2 \\ -1 \end{bmatrix} \right\}.
\]
5. Find a basis for the space $V \subset P_5$ of polynomials $p$ of degree up to 5 such that

$$p(-2) = \frac{dp}{dt}(-2) = \frac{d^2p}{dt^2}(-2) - \frac{d^3p}{dt^3}(-2) = 0.$$ 

What is the dimension of this space?