Date: December 22, 1997
Time: 15:00-17:30

NAME:..

STUDENT NO:..

DEPARTMENT: CS EE IE

MATH 220 MAKE-UP FINAL EXAM

IMPORTANT

1. This exam consists of 6 questions of equal weight.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.

Please do not write anything below this line.

1	2	3	4	5	6	TOTAL
20	20	20	20	20	20	120

1. Let $L\colon P_2 \to P_2$ be given by $p(t) \mapsto p(1) + 4p''(1)t + p'(1)t^2$. Find the eigenvalues and eigenvectors of L. Is L diagonalizable?

2. Show that there is no linear transformation $L\colon \mathbb{R}^5 \to \mathbb{R}^2$ such that

$$\operatorname{Ker} L = \{x \in \mathbb{R}^5 \mid x_1 = x_2,\ x_3 = x_4 = x_5\}.$$

3. Define the *trace* of a square $(n \times n)$-matrix $A = [a_{ij}]_{1 \le i,j \le n}$ to be $\operatorname{trace} A = \sum_{i=1}^n a_{ii}$. Prove or disprove:

1. $\operatorname{trace}(\alpha A) = \alpha \operatorname{trace} A$, $\alpha \in \mathbb{R}$;
2. $\operatorname{trace}(AB) = \operatorname{trace} A \cdot \operatorname{trace} B$;
3. $\operatorname{trace}(AB) = \operatorname{trace}(BA)$.

4. Let P_3 be the space of polynomials of degree up to 3 with the inner product

$$(p, q) = p(1) \cdot q(1) + p'(1) \cdot q'(1) + p''(1) \cdot q''(1) + p'''(1) \cdot q'''(1)$$

and $W = \operatorname{Span}\{1, t^2\}$. Find a basis for $W^\perp$.

5. Define an inner product on the space $M_{3,3}$ of (3×3)-matrices via $(A, B) = \operatorname{trace}(A^{\mathrm{T}} B)$ (see Problem 3 for the definition of trace). Use the Gramm-Schmidt process to orthonormalize the system

$$A_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

6. Evaluate

$$\begin{vmatrix} 1 & 2 & 2 & 0 & 3 \\ 0 & 1 & 2 & 0 & -1 \\ 2 & 3 & 5 & 2 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 3 & 3 & 0 & 4 \end{vmatrix}.$$