Solutions to Midterm I

Problem 1. Find the limits (without using l'Hôpital's rule):

(a) $\lim_{x\to 0} \dfrac{\cos^2 3x - 1}{\tan x \sin 2x}$.

(b) $\lim_{x\to 3} \dfrac{x^2 - 2x - 3}{x^3 - 2x^2 - 2x - 3}$.

Problem 2. Find the derivatives of the following functions:

(a) $y = \dfrac{(x^2+1)^3}{(x+1)^4}$.

(b) $y = \sin(x + \sqrt{x^2+1}) - \dfrac{1}{\pi + 1}$.

(c) $y = \left(\dfrac{\sin x}{\cos x - 1}\right)^3$.

Problem 3. Show that the tangent to the curve $y = x^3$ at any point (a, a^3) meets the curve again at a point where the slope of the tangent is four times the slope of the original tangent at (a, a^3).

Problem 4. Find y'' at the point $(x, y) = (1, -1)$, where y is a differentiable function of x satisfying the equation $x^2 + xy + y^2 = 1$.

Problem 5. A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. How fast is the shadow of the ball moving along the ground 0.5 sec later? (Assume that the ball falls a distance $s = 16t^2$ ft in t sec.)