ODTU-Bilkent Algebraic Geometry

Singular real plane sextic curves without real points

By

Alexander Degtyarev
(Bilkent)

Abstract: (joint with Ilia Itenberg)It is a common understanding that any reasonable geometric question about K3-surfaces can be restated and solved in purely arithmetical terms, by means of an appropriately defined homological type. For example, this works well in the study of singular complex sextic curves in P2 or quartic surfaces in P3 (see [1,2]), as well as in that of smooth real ones (see [4,6]). However, when the two are combined (both singular and real curves or surfaces), the approach fails as the `"obvious" concept of homological type does not fully reflect the geometry (cf., e.g., [3] or [5]).

We show that the situation can be repaired if the curves in question have empty real part or, more generally, have no real singular points; then, one can indeed confine oneself to the homological types consisting of the exceptional divisors, polarization, and real structure.

Still, the resulting arithmetical problem is not quite straightforward, but we manage to solve it and obtain a satisfactory classification in the case of empty real part; it matches all known results obtained by an alternative purely geometric approach. In the general case of smooth real part, we also have a formal classification; however, establishing a correspondence between arithmetic and geometric invariants (most notably, the distribution of ovals among the components of a reducible curve) still needs a certain amount of work.

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