# TOPOLOGY SEMINAR

# Fundamental groups of fusion systems

**By**


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**Abstract:** Fix a prime p. The fusion system of a finite group G with respect to a Sylow p-subgroup S of G is the category $F_S(G)$ whose objects are the subgroups of S, and whose morphisms are the homomorphisms induced by conjugation in G.
More generally, an abstract fusion system over a p-group S is a category F whose objects are the subgroups of S and whose morphisms are injective homomorphisms between the subgroups that satisfy certain axioms motivated by the Sylow theorems.
The geometric realization |F| of a fusion system F is (roughly) the cell complex with one vertex for each object in F, one edge for each morphism (attached to its source and target vertices), one 2-simplex for each commutative triangle of morphisms, etc. The space |F| itself is contractible (as is the realization of every category with initial object), and hence not very interesting to an algebraic topologist.
However, the realizations of certain full subcategories of F, and in particular their fundamental groups, do have important applications. For example, when $F^c \subseteq F$ denotes the full subcategory of F-centric subgroups of S (very roughly, those that contain their centralizers in S), the group $\pi_1(|F^c|)$ can be used to classify certain fusion subsystems of F. More recently, when $F = F_S(G)$, for finite G and $S \in Syl_p(G)$, and $F^* \subseteq F$ is the full subcategory whose objects are the nontrivial subgroups $1 < P \leq S$, then $\pi_1(|F^*|)$ plays a key role in work by Grodal to describe the group of "Sylow trivial" kG-modules when k is a field of characteristic p.
We will describe these applications, and then give some examples of calculations that have been made of these and other fundamental groups.


**Date:** 8 June, 2020
**Time:** 13:30 – 14:30
**Place:** ZOOM. To request the event link, please send a message to matthew.gelvin@bilkent.edu.tr