TOPOLOGY SEMINAR

Realizability of fusion systems

By

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Abstract: Fix a prime p, and let Z/p^∞ be the union of the cyclic p-groups $Z/p < Z/p2 < Z/p3 < \dots$. Thus Z/p^∞ is isomorphic to the group of all complex roots of unity of p-power order. A group S is discrete p-toral if it contains a normal subgroup S0 of p-power index such that S0 is isomorphic to $(Z/p^\infty)r$ for some $r \geq 0$.

A saturated fusion system over a discrete p-toral group S is a category whose objects are the subgroups of S, and whose morphisms are homomorphisms between the subgroups satisfying certain axioms. For example, if G is a finite group and S is its Sylow p-subgroup, the fusion system of G over S is the category FS(G) whose objects are the subgroups of S and whose morphisms are those homomorphisms induced by conjugation by elements of G. If G is a compact Lie group, then it contains a maximal discrete p-toral subgroup $S \leq G$ that is unique up to conjugacy, and FS(G) is defined in exactly the same way. In all of these cases, the fusion system FS(G) is saturated.

These were the motivating examples for studying saturated fusion systems: first over finite p-groups and then over discrete p-toral groups. But in addition, in my work with Ran Levi and Carles Broto, we found a large class of infinite discrete groups, including all linear torsion groups, that have Sylow p-subgroups (unique up to conjugacy) and saturated fusion systems. Here, "linear" means that $G \leq GLn(K)$ for some $n \geq 1$ and some field K with characteristic different than p, and "torsion" means that all elements have finite order.

When F is a saturated fusion system over a finite p-group S, it is natural to say that F is "realizable" if it is isomorphic to the fusion system of a finite group, and "exotic" otherwise. But it is less obvious what realizable should mean when applied to fusion systems over infinite discrete p-toral groups (realizable by what?). Moreover, until recently, we had no tools for proving that certain fusion systems (finite or infinite) cannot be realized as fusion systems of linear torsion groups.

In recent work (still in progress) with Ran Levi and Carles Broto, we have been looking at this question, and have found tools for proving that certain fusion systems cannot be realized by linear torsion groups. For example, we showed that if F is a fusion system over a finite p-group and is realized by an infinite linear torsion group, then it is realized by a finite group. We also went through lists of known saturated fusion systems over infinite discrete p-toral groups, and found many examples that are exotic and many that are realizable.

Date: Monday, April 17, 2023
Time: 13:30
Place: SA141 - Mathematics Seminar Room & ZOOM
To request the event link, please send a message to cihan.okay@bilkent.edu.tr