Written Exam: The duration of the exam is three hours. There are four subject areas. In each subject area, there are four questions. The candidate is to attempt two questions from each subject area. The syllabi for the four subject areas are as below.

Oral Exam: The candidate is to make a fifteen-minute presentation directed as if towards a general audience. This is to be followed by about fifteen minutes of questions. (The questions are likely to be requests for pedagogical clarification of basic concepts.) The assessment will be for teaching and communication. No credit will be awarded for specialist erudition.

ALGEBRA

Any overlap with lecture courses is accidental. The candidate is expected to master the material through independent study (using multiple sources). For theorems marked with a ⋆, the candidate is expected to be able to state and apply the theorem, but proof is not demanded.

Set theory: ⋆ Zorn's Lemma, ⋆ existence and uniqueness of the algebraic closure of a field.


Galois Theory: ⋆ Eisenstein’s Criterion, ⋆ Artin’s Primitive Element Theorem, ⋆ Fundamental Theorem of Galois Theory, ⋆ Unsolvability of the Quintic. Calculation of Galois groups for polynomials such as \( X^6 - 2 \).


Finite Group Representations: Frobenius Reciprocity. Maschke’s Theorem. Orthogonality relations. Construction of ordinary character tables for groups such as \( S_4 \) and \( A_5 \).

Pervasive bibliography:

Concentrated bibliography:

Ambient bibliography:
ANALYSIS

MATH 501 – REAL ANALYSIS I

1. Algebra of sets. Borel sets and Bair functions.
4. Integration. Limit theorems for Lebesgue integral.
6. Spaces of integrable functions.

Books:

MATH 502 – REAL ANALYSIS II

5. Weak Topologies.

Books:

COMPLEX ANALYSIS

A. Holomorphic Functions. Cauchy-Riemann equations, multi-valued functions, Cauchy theorem and integral formula, Morera theorem, power series representation, sequences of holomorphic functions, uniqueness theorem, open mapping theorem, maximum modulus principle, Cauchy theorem for multiply connected regions, Cauchy-type integrals.

B. Singularities. Classification, Laurent series, residue theorem, argument principle, Rouché theorem, residues at infinity, evaluation of integrals and sums.

C. Conformal Mapping. Preservation of angles, Schwarz-Pick lemma, mapping by Möbius transformations, normal families, Riemann mapping theorem, continuity at the boundary, simply connected regions.

D. Meromorphic and Entire Functions. Runge theorem on approximation by rational functions, Mittag-Leffler theorem, infinite products, Weierstrass factorization theorem, Jensen formula, Blaschke products, gamma and zeta functions.
E. **Analytic Continuation.** Regular and singular points, gap series, Schwarz reflection principle, continuation along curves, monodromy theorem.

F. **Harmonic Functions.** Maximum principle, mean value property, Harnack theorem, Poisson integral.

**Books:**

(Chapters 10–16 of [1] covers most of the topics, but the sections 11.15–11.32, 12.11–12.14, 14.10–14.15, 15.25–15.27, 16.17–16.22 are less relevant. The other references also contain sections more or less equivalent to these. For a few topics not covered by the above (such as multi-valued functions, residues at infinity, gamma and zeta functions), one can consult the other references.)

APPLIED MATHEMATICS

MATH 543 Methods of Applied Mathematics I


4. Calculus of variations, necessary condition, Euler–Lagrange equations, Lagrange functions depending on higher derivatives, null Lagrange functions, lagrange function of given DE. Lagrange function with several dependent variables. Iso-perimetric problems.


For more details see the course web site http://www.fen.bilkent.edu.tr/gurses/applied1.html.

For Qualifying Exam questions, see those assigned exercises at this web site that are related to the above subjects.

MATH 544 Methods of Applied Mathematics II

1. Partial differential equation models (G–L, Copson, Logan). Classification of 2nd order partial differential equations (G–L, Copson) The Cauchy–Kowalewsky theorem (Copson). Linear, quasi-linear, half-linear equations (Copson). Initial and BV problems for the wave equation (G–L). Some existence and uniqueness theorems (G–L). First order hyperbolic systems. The Riemann Method (G–L, Copson). Exact solutions, uniqueness, maximum-minimum theorems of hyperbolic type of (the wave) equation, of parabolic type of (the heat) equation, of elliptic type of (the Laplace) equation (G–L, D–K).


DK = Dennery and Krzywicki, G–L = Roland B Guenter and J W Lee.

For more details see the web site http://www.fen.bilkent.edu.tr/gurses/applied2.html.

For Qualifying Exam questions, see those assigned exercises at this web site that are related to the above subjects.
GEOMETRY AND TOPOLOGY

A. Geometry

1. Differentiable Manifolds, Differentiable Functions and Mappings: Differentiable manifolds, differentiable functions and mappings, rank of a mapping, immersions, submersions, submanifolds and embeddings.

2. Vector Fields on Manifolds: Tangent space at a point of a manifold, the differential of a differentiable mapping, vector fields, Lie bracket of vector fields.

3. Tensors and Tensor Fields on Manifolds: Tensors, tensor fields and differential forms, pull-back of a differentiable mapping by differentiable mapping, exterior differentiation, Riemannian metric on manifolds, orientation on manifolds, volume element.

4. Integration on Manifolds: Integration on manifolds, manifolds with boundary, boundary orientation of the boundary of a manifold, Stokes’s Theorem.

References:
Boothby, W., An Introduction to Differentiable manifolds and Riemannian Geometry (sections: III.1-III.5, IV.1-IV.2, V.1-V.8, VI.1, VI.2, VI.4, VI.5)

B. Topology

1. Topological spaces and continuous functions: Topological spaces, basis and subbasis, subspace topology, continuous functions, product topology, metric topology, quotient topology.

2. Compactness: Compact spaces, compact sets in Rn, Heine–Borel Theorem, Tychonoff Theorem, limit-point compactness, sequential compactness, compactness in metric spaces, local compactness and one-point compactification.

3. Connectedness: Connected spaces, path-connected spaces, components, local connectedness, local path-connectedness.

4. Separation and Countability Properties: T0, Hausdorff, regular, normal spaces; Uryshon Lemma, Tietze Extension Theorem, countability properties; Lindelf, separable, countably compact spaces

References:
Munkres J., Topology, a First Course, (sections: 2.1-2.10, 3.1-3.8, 4.1- 4.3)
Willard S., General Topology, (sections: 2, 3, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19. In section 19 only one-point compactification is included.)

C. Algebraic Topology:


D. Algebraic Geometry (Math 551):

**Theory of algebraic varieties:** Affine and projective varieties, dimension, singular points, divisors, differentials, Bezout’s theorem.

Main Reference: R. Hartshorne, *Algebraic Geometry*, (Chapter 1).


E. Differential Geometry I (Math 545):

Lie derivative of tensor fields. Connections, covariant differentiation of tensor fields, parallel translation, holonomy, curvature, torsion.

