Bilkent University
Department of Mathematics

Problem Of The Month

Term: April 2024

Let $n \geq 3$ be an integer and $a_1, a_2, \dots, a_n$ be real numbers. For each $1 \leq k \leq n$ the real numbers $b_1, b_2, \dots, b_{n+1}$ are defined by

$$b_k = \frac{a_k + \max\{a_{k+1}, a_{k+2}\}}{2}$$

and $b_{n+1} = b_1$ ($a_{n+1} = a_1$ and $a_{n+2} = a_2$). Show that the inequality

$$\sum_{i=1}^{n} (a_i - a_{i+1})^2 \geq \sum_{i=1}^{n} (b_i - b_{i+1})^2$$

is held for each $n \geq 3$ and all real numbers $a_1, a_2 \dots, a_n$.