



Bilkent University
Department of Mathematics

PROBLEM OF THE MONTH

Term: March 2024

For each positive integer n , let $\varphi(n)$ be the total number of positive integers not exceeding n and co-prime with n . Find all functions $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ satisfying the following two conditions:

$$a - b \mid f(a) - f(b) \quad \text{and}$$

$$f(\varphi(a)) = \varphi(f(a)).$$