Problem:

Find the smallest value of

\[ xy^3z^2 + \frac{4z}{x} - 8yz - \frac{4}{xy} \]

where \( x, y, z \) are positive real numbers satisfying at least one of the following inequalities:

\[ xy > \frac{1}{2} \quad \text{and} \quad yz > 1. \]

Solution: Answer: \(-8\). This value is attained at \( xy = 1 \) and \( yz = 2 \).

By AM-GM inequality we get

\[ 2y - \frac{1}{x} \leq xy^2 \quad \text{and} \quad 4z - \frac{4}{y} \leq yz^2 \]  \hspace{1cm} (1)

Since at least one of the inequalities: \( xy > \frac{1}{2} \) and \( yz > 1 \) is held at least one of the inequalities

\[ 2y - \frac{1}{x} > 0 \quad \text{and} \quad 4z - \frac{4}{y} > 0 \]

is also held. Therefore, we can multiply two inequalities in (1) and

\[ -8 - \frac{4z}{x} + 8yz + \frac{4}{xy} = (2y - \frac{1}{x})(4z - \frac{4}{y}) \leq xy^3z^2. \]

Thus,

\[ xy^3z^2 + \frac{4z}{x} - 8yz - \frac{4}{xy} \geq -8. \]