



Bilkent University  
Department of Mathematics

## PROBLEM OF THE MONTH

**Term:** November 2022

For each pair  $(p, a)$ , where  $p$  is a prime number and  $a$  is a positive integer the sequences  $\{a_n\}$  and  $\{b_n\}$  are defined as

$$a_1 = a \text{ and } a_{n+1} = a_n + p \lfloor \sqrt[p]{a_n} \rfloor$$

$$b_n = \sqrt[p]{a_n}$$

A pair  $(p, a)$  is said to be *good* if the sequence  $\{b_n\}$  contains infinitely many integers. Find all values of  $p$  such that all pairs  $(p, a)$ ,  $a = 1, 2, \dots$  are good.