

Bilkent University Department of Mathematics

PROBLEM OF THE MONTH

Term: June 2022

Let \mathbb{Q} and \mathbb{Q}^+ be the set of all rational and all positive rational numbers, respectively. Find all functions $f: \mathbb{Q}^+ \to \mathbb{Q}$ satisfying

$$f(x) + f(y) = \left(f(x+y) + \frac{1}{x+y}\right)(1 - xy + f(xy))$$

for all $x, y \in \mathbb{Q}^+$.