Bilkent University
Department of Mathematics

Problem Of The Month

Term: April 2021

There are two boxes one coloured red and other coloured white. At the beginning each box contains 500 beads and there are infinitely many spare beads. Alice and Bob play a game alternatively making moves, Alice making first move. A player making a move performs one of the four actions: adds a bead to the red box (R^+), removes a bead from the red box (R^-), adds a bead to the white box (W^+), removes a bead from the white box (W^-) depending on the last move of her/his opponent and according to the following table: (at the first move Alice can choose any one of the four actions):

Last move	R^+	R^-	W^+	W^-
Alice's move	R^+ or W^+	R^- or W^-	W^+ or R^-	W^- or R^+
Bob's move	R^+ or W^-	R^- or W^+	W^+ or R^+	W^- or R^-

A player making 1001 or 0 beads in any box loses the game. Who wins?

Note: Due to the table after Bob's move R^+ Alice can move either R^+ or W^+, after Alice's move R^+ Bob can move either R^+ or W^-, ...