Bilkent University
Department of Mathematics

PROBLEM OF THE MONTH

Term: July - August 2019

For positive integer k, let

$$R_n = \{-k, -(k-1), \ldots, -1, 1, \ldots, k-1, k\} \text{ for } n = 2k \text{ and}$$
$$R_n = \{-k, -(k-1), \ldots, -1, 0, 1, \ldots, k-1, k\} \text{ for } n = 2k+1.$$

A device consists of several balls and red or white ropes connecting some ball pairs. A *labeling* is a coloring of each ball by one of the elements of R_n. We say that a labeling is *good* if colors of any two connected balls are different. We say that a labeling is *sensitive* if the colors of any two balls connected by white rope are different and the sum of colors of any two balls connected by red rope is not equal to 0.

Let $n \geq 3$ be fixed. Suppose that any device which has a good labeling by R_n has also a sensitive labeling by R_m. Find the smallest possible value of $m = m(n)$.