Problem: Show that for each pair of positive integers \((a, b)\) there is a positive integer \(n\) such that \(n^2 + an + b\) has at least 2018 distinct prime divisors.

Solution: By induction over \(k\) we will prove that for each pair of positive integers \((a, b)\) there is a positive integer \(n\) such that \(n^2 + an + b\) has at least \(k\) distinct prime divisors. The case \(k = 1\) is obvious. For given \((a, b)\) suppose that for some \(n_k\) \(n_k^2 + an_k + b\) has \(k\) distinct prime divisors \(p_1, p_2, \ldots, p_k\). Then \(p_1p_2\ldots p_k\) divides \(m = n_k^2 + an_k + b\). Let us define \(n_{k+1} = n_k(m^2 + 1)\). Then

\[
\begin{align*}
n_{k+1}^2 + an_{k+1} + b &= n_k^2(m^2 + 1)^2 + an_k(m^2 + 1) + b \\
&= n_k^2(m^4 + 2m^2 + 1) + an_k(m^2 + 1) + b = n_k^2m^4 + 2m^2n_k^2 + an_km^2 + m \\
&= m(n_k^2m^3 + 2mn_k^2 + an_km + 1).
\end{align*}
\]

The last expression is divisible by \(p_1, p_2, \ldots, p_k\) and some another prime number \(p_{k+1}\) since \(m\) and \(n_k^2m^3 + 2mn_k^2 + an_km + 1\) are relatively prime. The inductive hypothesis for \(k + 1\) is proved. Done.