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PROBLEM OF THE MONTH

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Problem: In a country consisting of 2015 cities between any two cities there is exactly one direct round flight operated by some air company such that direct flights between any three cities are operated by three different air companies. What is the possible minimal number of air companies in the country?

Solution: The answer is 2015. Since there are 1007 disjoint pairs of cities, any air company can operate at most between 1007 pairs. There are $\binom{2015}{2}$ direct flights, therefore the number of air companies is at least $\frac{2015 \cdot 2014}{2 \cdot 1007} = 2015$.

Example for 2015 air companies:

Let us label the cities by $1, 2, \dots, 2015$ and connect the cities i and j by a company labeled $(i + j) \pmod{2015}$.