Problem:

Let \( d(n) \) be the smallest prime divisor of integer \( n \not\in \{0, -1, +1\} \). Determine all polynomials \( P(x) \) with integer coefficients satisfying

\[
P(n + d(n)) = n + d(P(n))
\]

for all integers \( n > 2014 \) for which \( P(n) \not\in \{0, -1, +1\} \).

Solution:

The answer: \( P(x) = x, P(x) \equiv 1, 0, -1 \).

We start with the case when \( \text{deg}(P(x)) \geq 2 \). Let us take \( n = q \), where \( q \) is prime: \( P(q + d(q)) = q + d(P(q)) \) yields \( P(2q) = q + d(P(q)) \). Therefore, \( |P(2q)| \leq q + |P(q)| \) and

\[
\frac{|P(2q)|}{|P(q)|} \leq q + 1 \quad (\dagger)
\]

Now when \( q \) increases the left hand side of (\dagger) goes to \( 2^{\text{deg}(P(x))} \), but right hand side goes to 1. Contradiction.

Now let \( \text{deg}(P(x)) = 1 \) and \( P(x) = bx + c \). Then again for \( n = q \) we get \( 2bq + c = q + d(bq + c) \) and \( (2b - 1)q + c = d(bq + c) \). If \( q \) is sufficiently large we get that \( b \geq 1 \) and \( (2b - 1)q + c \leq bq + c \) which in turn yields \( b = 1 \). Thus, \( n + d(n) + c = n + d(n + c) \) and
\[ d(n) + c = d(n + c) \quad (††) \]

If \( c > 0 \) then for \( n = 2^l - c \) the left hand side of (††) is at least 3, while the right hand side of (††) is 2.

If \( c < 0 \) then for \( n = 2^l \) the left hand side of (††) is at most 1, while the right hand side of (††) is at least 2.

Thus, \( c = 0 \) and \( P(x) = x \).

If \( \text{deg}(P(x)) = 0 \) then for \( c \neq 0, \pm 1 \) we get \( c = n + d(c) \), a contradiction.