Bilkent University
Department of Mathematics

Problem Of The Month

June 2012

Problem:

In how many ways the set $\{1, 2, \ldots, 2012\}$ can be partitioned into two subsets so that for any two distinct elements a and b belonging to the same subset $a + b \neq 3^k, k = 1, 2, \ldots$.

Solution:

Let us distribute the numbers $1, 2, \ldots, 2012$ ascending order. 1 and 2 should be placed into different sets. It can be readily observed that for $k = 1, 2, \ldots, 6$

if a number p belongs to the interval $[3^k, \frac{3^{k+1}}{2})$ then it can be placed in any of the two sets and

if a number p belongs to the interval $(\frac{3^{k+1}}{2}, 3^{k+1} - 1]$ then it should be placed into the set not containing $3^{k+1} - p$.

Therefore, the numbers p which can be placed in any of the two sets are integers belonging to the intervals $[3, 4], [9, 13], [27, 40], [81, 121], [243, 364], [729, 1093]$. Thus, the total number of cases when there are 2 possibilities is $2 + 5 + 14 + 41 + 122 + 365 = 549$ and the answer is 2^{549}.