Problem: Find the smallest real number $A$ such that for all triangle angles $\alpha, \beta$ and $\gamma$ the inequality
\[ \sin^2 \alpha + \sin^2 \beta - \cos \gamma \leq A \]
holds.

Solution:
The answer is $\frac{5}{4}$.

Let us prove that
\[ f = \sin^2 \alpha + \sin^2 \beta - \cos \gamma < \frac{5}{4} \]
Indeed, $f = 2 - \cos^2 \alpha - \cos^2 \beta - \cos \gamma = 1 - \frac{1}{2}(\cos 2\alpha + \cos 2\beta) + \cos(\alpha + \beta)$
$= 1 - (\cos(\alpha + \beta))(\cos(\alpha - \beta) - 1)$
and the inequality (1) is equivalent to
\[ (\cos(\alpha + \beta))(1 - \cos(\alpha - \beta)) < \frac{1}{4} \]
(2) follows from the inequality $ab \leq \frac{(a+b)^2}{4}$. Indeed, 
$(\cos(\alpha + \beta))(1 - \cos(\alpha - \beta)) \leq \frac{1}{4}(\cos(\alpha + \beta) + 1 - \cos(\alpha - \beta))^2 = \frac{1}{4}(1 - 2 \sin \alpha \sin \beta)^2 < \frac{1}{4}$, since $0 < \sin \alpha \sin \beta < 1$ ($\alpha, \beta$ are triangle angles).

(1) is proved. Now note that if $\gamma = \frac{2\pi}{3}$ and $\alpha$ approaches to $\frac{\pi}{3}$, then $f$ approaches to $\frac{5}{4}$. Done.