Bilkent University
Department of Mathematics

Problem Of The Month

June 2007

Problem: Determine all prime numbers p such that the total number of positive divisors of $A = p^2 + 1007$ (including 1 and A) is less than 7.

Solution: The answer is: $p = 2$.

If $p = 2$, then $A = 1011 = 3 \cdot 337$ has 4 divisors.
If $p = 3$, then $A = 2^3 \cdot 127$ has 8 divisors.
If $p > 3$, then $A = p^2 - 1 + 1008 = (p-1)(p+1) + 24 \cdot 42$. But $(p-1)(p+1)$ is divisible by $2 \cdot 4$ and 3. Therefore, A has at least 7 divisors: $1, 2, 3, \frac{A}{6}, \frac{A}{3}, \frac{A}{2}, A$.