Problem: Suppose that $A_1, A_2, \ldots, A_n$ are points on the plane located in such a way that for any point $P$ on the same plane at least one of the distances $\text{dist}(P, A_i)$, $i = 1, 2, \ldots, n$ is irrational. Find the minimal possible value of $n$.

Solution: The answer is 3.

Consider a line $L$ perpendicular to the line segment $[A_1A_2]$ and passing through the center of the segment $[A_1A_2]$. It is clear that there are infinitely many points $P$ located on $L$ such that the distance between $P$ and $A_1$ (so between $P$ and $A_2$) is rational. Therefore, $n \geq 3$. Let us show that $n = 3$.

Let $A_1$ and $A_2$ be two points on the plane with $\text{dist}(A_1, A_2) = \sqrt{2}$ and $A_3$ be the center of $[A_1A_2]$. Let us show that for any point $P$ on the same plane at least one of the distances $\text{dist}(P, A_i)$, $i = 1, 2, 3$ is irrational. If $P$ lies on the line passing through $A_1$ and $A_2$, then obviously one of these distances is irrational. Suppose that $P$ does not belong to this line. Consider the parallelogram with vertices $A_1, P, A_2$ and $Q$ ($Q$ is uniquely determined by $A_1, P, A_2$). Then

$$|A_1A_2|^2 + |PQ|^2 = 2|PA_1|^2 + 2|PA_2|^2,$$

or

$$|A_1A_2|^2 = 2|PA_1|^2 + 2|PA_2|^2 - 4|PA_3|^2.$$  \hfill(*)

Since $|A_1A_2|^2 = (\sqrt{2})^2 = \sqrt{2}$ is irrational, at least one of the terms in (*) is irrational. Therefore, at least one of the distances $|PA_i|$, $i = 1, 2, 3$ is irrational. Done.