In This Lecture:

- Interband Transitions in Bulk Se/c
  - Momentum Matrix Element
  - Polarization dependence
- Interband Transitions in Quantum Wells
- Intraband Transitions in Bulk & QWs
Interband Transitions in Bulk Se/c

As the photon momentum is negligible compared to electronic crystal momenta, the transitions are (almost) vertical ($k_{op}=0$)

[See, next page]

For parabolic bands:

$$\hbar \omega = (E_e - E_v) + \frac{\hbar^2 q^2}{2 \left( \frac{1}{m_e} + \frac{1}{m_h} \right)}$$

Figure 9.5: The positions of the electron and hole energies at vertical k-values. The electron and hole energies are determined by the photon energy and the carrier masses. Since the photon momentum is negligible the transitions are vertical.

Ref: Singh
**Momentum Matrix Elements**

Predominantly we are interested in transitions between CB and VB so that \( i \rightarrow v, \ f \rightarrow c \)

\[
\hat{e} \cdot \vec{p} = \hat{e} \cdot \int \psi^*_{c,k_c} e^{i\vec{k}_{op} \cdot \vec{r}} \vec{p} \psi_{v,k_v} \, d^3r
\]

where

\[
\psi_{c,k_c}(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k}_{c} \cdot \vec{r}} u_{c,k_c}(\vec{r}); \quad \text{similarly for } \psi_{v,k_v}(\vec{r})
\]

vanishes when \( k_{op} = 0 \) due to Bloch fn orthogonality

\[
\vec{p}_{cv} = \int u^*_{c,k_c}(\vec{r}) e^{-ik_c \cdot \vec{r}} e^{i\vec{k}_{op} \cdot \vec{r}} \hbar k_v e^{ik_v \cdot \vec{r}} u_{v,k_v}(\vec{r}) \, \frac{d^3r}{V}
\]

+ \[
+ \int u^*_{c,k_c}(\vec{r}) e^{-ik_c \cdot \vec{r}} e^{i\vec{k}_{op} \cdot \vec{r}} \left( \frac{\hbar}{i} \nabla u_{v,k_v}(\vec{r}) \right) e^{ik_v \cdot \vec{r}} \, \frac{d^3r}{V}
\]

\[
\begin{cases}
\vec{p}_{cv} = \int u^*_{c,k_c}(\vec{r}) \left( \frac{\hbar}{i} \nabla u_{v,k_v}(\vec{r}) \right) \frac{d^3r}{\Omega} \\
\int e^{i(-\vec{k}_c + \vec{k}_v + \vec{k}_{op}) \cdot \vec{r}} \, \frac{d^3r}{V} \end{cases}
\]

Treat slowly-varying (envelopes) and the cell-periodic parts separately

\[
\vec{p}_{cv} = \delta_{\vec{k}_c + \vec{k}_v + \vec{k}_{op}} \int u^*_{c,k_c}(\vec{r}) \left( \frac{\hbar}{i} \nabla u_{v,k_v}(\vec{r}) \right) \frac{d^3r}{\Omega}
\]

unit cell volume

xtal volume
Electric dipole forbidden transitions

When certain $p_{cv}$ transition matrix element vanishes (due to some symmetry reason etc.) this is termed as an electric dipole-forbidden-transition. In this case higher-order contributions such as electric quadrupole and magnetic dipole transitions become important. Compared to the electric dipole transitions they are reduced in strength by a factor of $(\text{lattice constant/wavelength of light})^2$, that requires very high frequencies (UV to X-rays)...
Polarization Dependence

Recall se/c band edge states:

Conduction Band:
\[
|i_S \uparrow\rangle, |i_S \downarrow\rangle
\]

Valence Bands (only HH, LH):
\[
\begin{align*}
|\frac{3}{2}, \frac{3}{2}\rangle &= \frac{-1}{\sqrt{2}} |(X + iY) \uparrow\rangle, \\
|\frac{3}{2}, -\frac{3}{2}\rangle &= \frac{1}{\sqrt{2}} |(X - iY) \downarrow\rangle, \\
|\frac{3}{2}, \frac{1}{2}\rangle &= \frac{-1}{\sqrt{6}} |(X + iY) \downarrow\rangle + \sqrt{\frac{2}{3}} |Z \uparrow\rangle, \\
|\frac{3}{2}, -\frac{1}{2}\rangle &= \frac{1}{\sqrt{6}} |(X - iY) \uparrow\rangle + \sqrt{\frac{2}{3}} |Z \downarrow\rangle
\end{align*}
\]

Momentum-matrix parameter:

\[
P_z = \langle iS | p_z | X \rangle = \langle iS | p_y | Y \rangle = \langle iS | p_z | Z \rangle = \frac{m_0}{\hbar} P
\]

CB to HH Transitions:
\[
\begin{align*}
\langle iS \uparrow | \vec{p} | \frac{3}{2}, \frac{3}{2}\rangle &= -\frac{P_z}{\sqrt{2}} (\hat{x} + i\hat{y}), \\
\langle iS \downarrow | \vec{p} | \frac{3}{2}, \frac{3}{2}\rangle &= 0,
\end{align*}
\]

\[
\begin{align*}
\langle iS \downarrow | \vec{p} | \frac{3}{2}, -\frac{3}{2}\rangle &= \frac{P_z}{\sqrt{2}} (\hat{x} - i\hat{y}), \\
\langle iS \uparrow | \vec{p} | \frac{3}{2}, -\frac{3}{2}\rangle &= 0.
\end{align*}
\]

\[
\begin{align*}
\langle iS \uparrow | \vec{p} | \frac{3}{2}, \frac{1}{2}\rangle &= P_z \sqrt{\frac{2}{3}} \hat{z}, \\
\langle iS \downarrow | \vec{p} | \frac{3}{2}, \frac{1}{2}\rangle &= -\frac{P_z}{\sqrt{6}} (\hat{x} + i\hat{y}), \\
\langle iS \downarrow | \vec{p} | \frac{3}{2}, -\frac{1}{2}\rangle &= P_z \sqrt{\frac{2}{3}} \hat{z}, \\
\langle iS \uparrow | \vec{p} | \frac{3}{2}, -\frac{1}{2}\rangle &= \frac{P_z}{\sqrt{6}} (\hat{x} - i\hat{y}).
\end{align*}
\]

WATCH OUT:
No coupling of the z-polarized light between CB & HH
Reflections on the polarization dependence

- For a cubic xtal what differentiates $z$ from $x$ or $y$?
- Recall that in defining the expansion basis vectors we assumed electron wavevector to be along $z$ direction
- For that reason we are also using the $z$-projection of the spin

- Does that give enough support for singling out $z$ from $x$ or $y$ direction?
- After all that’s just for the sake of formulation, say a convention
- Away from $k=0$ HH & LH become mixed
- So only at $k=0$ we could talk about such a selectivity
- But at $k=0$ we lose any sense of direction of the $k$-vector!
To Remind you the LK Hamiltonian

\[
\mathbf{H}^{\text{LK}} = - \begin{bmatrix}
P + Q & -S & R & 0 & -S/\sqrt{2} & \sqrt{2} R \\
-S^+ & P - Q & 0 & R & -\sqrt{2} Q & \sqrt{3}/2 S \\
R^+ & 0 & P - Q & S & \sqrt{3}/2 S^+ & \sqrt{2} Q \\
0 & R^+ & S^+ & P + Q & -\sqrt{2} R^+ & -S^+/\sqrt{2} \\
-S^+/\sqrt{2} & -\sqrt{2} Q^+ & \sqrt{3}/2 S & -\sqrt{2} R & P + \Delta & 0 \\
\sqrt{2} R^\ast & \sqrt{3}/2 S^+ & \sqrt{2} Q^+ & -S/\sqrt{2} & 0 & P + \Delta
\end{bmatrix}
\]

complex conjugate

where

\[
\begin{align*}
P &= \frac{\hbar^2 \gamma_1}{2m_0} \left( k_x^2 + k_y^2 + k_z^2 \right) \\
Q &= \frac{\hbar^2 \gamma_2}{2m_0} \left( k_x^2 + k_y^2 - 2k_z^2 \right) \\
R &= \frac{\hbar^2}{2m_0} \left[ -\sqrt{3} \gamma_2 (k_x^2 - k_y^2) + i2\sqrt{3} \gamma_3 k_x k_y \right] \\
S &= \frac{\hbar^2 \gamma_3}{m_0} \sqrt{3} (k_x - i k_y) k_z
\end{align*}
\]
Averaging over the polarization for bulk

- These considerations suggest us to consider unpolarized light.

- Equivalently we shall consider electron wavevector to point along a general direction and average the matrix element over the solid angle.

Let the electron wavevector to be along a direction \((\theta, \Phi)\):

\[ \mathbf{k} = k \sin \theta \cos \phi \, \hat{x} + k \sin \theta \sin \phi \, \hat{y} + k \cos \theta \, \hat{z} \]

For illustration consider CB-HH transition:

\[ |\hat{e} \cdot \mathbf{p}_{c\nu}|^2 \equiv \langle |\hat{e} \cdot \mathbf{M}_{c-nh}|^2 \rangle = \frac{1}{4\pi} \int |\hat{x} \cdot \mathbf{M}_{c-nh}|^2 \sin \theta \, d\theta \, d\phi \]

averaging over the solid angle.
\[ \begin{align*}
\text{CB:} & \quad |iS \downarrow \rangle \quad \text{and} \quad |iS \uparrow \rangle \\
\left| \begin{array}{c}
\frac{3}{2}, \frac{3}{2} \\
\frac{3}{2}, -\frac{3}{2}
\end{array} \rightangle & = -\frac{1}{\sqrt{2}} \left( X' + iY' \right) \uparrow \\
& = -\frac{1}{\sqrt{2}} \left( \cos \theta \cos \phi - i \sin \phi \right) X
\end{align*} \]

\[ \begin{align*}
\text{HH:} & \quad \left| \begin{array}{c}
\frac{3}{2}, -\frac{3}{2} \\
\frac{3}{2}, \frac{3}{2}
\end{array} \rightangle = \frac{1}{\sqrt{2}} \left( X' - iY' \right) \downarrow \\
& = \frac{1}{\sqrt{2}} \left( \cos \theta \cos \phi + i \sin \phi \right) X \\
& + (\cos \theta \sin \phi + i \cos \phi) Y - \sin \theta Z \downarrow
\end{align*} \]

Note that for ease of calculation we keep the spin parts in the new (rotated) coordinate system...

Ref: Chuang
\[
\langle iS^{\uparrow}\mid p_{\frac{3}{2}}, \frac{3}{2}\rangle' = -\left[(\cos \theta \cos \phi - i \sin \phi) \hat{x} + (\cos \theta \sin \phi + i \cos \phi) \hat{y} - \sin \theta \hat{z}\right]\frac{P_x}{\sqrt{2}}
\]

\[
\langle iS^{\downarrow}\mid p_{\frac{3}{2}}, -\frac{3}{2}\rangle' = \left[(\cos \theta \cos \phi + i \sin \phi) \hat{x} + (\cos \theta \sin \phi - i \cos \phi) \hat{y} - \sin \theta \hat{z}\right]\frac{P_x}{\sqrt{2}}
\]

\[
\langle iS^{\uparrow}\mid p_{\frac{3}{2}}, -\frac{3}{2}\rangle = 0
\]

\[
\langle iS^{\downarrow}\mid p_{\frac{3}{2}}, \frac{3}{2}\rangle = 0
\]

Consider, for instance optical transition from the CB of one spin, say \(\langle iS^{\uparrow}\mid p_{\frac{3}{2}}, \frac{3}{2}\rangle\), to either of the HH bands \(\langle \frac{3}{2}, \frac{3}{2}\rangle', \frac{3}{2}, -\frac{3}{2}\rangle\); one of them is already zero.

Ref: Chuang
**Bulk Momentum Matrix Element for Unpolarized Light**

\[
|\hat{e} \cdot \mathbf{p}_{cv}|^2 = \left| \langle \hat{e} \cdot \mathbf{M}_{c \rightarrow h} \rangle \right|^2 = \frac{1}{4\pi} \int |\hat{e} \cdot \mathbf{M}_{c \rightarrow h}|^2 \sin \theta \, d\theta \, d\phi
\]

\[
= \frac{1}{4\pi} \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\phi (\cos^2 \theta \cos^2 \phi + \sin^2 \phi) \frac{P_x^2}{2}
\]

\[
= \frac{1}{3} P_x^2 = M_b^2
\]

where \( M_b^2 = \frac{1}{3} P_x^2 = \frac{m_0^2}{3\hbar^2} P^2 \)  

Kane’s parameter, (not a surprise)

\[
= \left( \frac{m_0}{m_e^*} - 1 \right) \frac{m_0 E_g (E_g + \Delta)}{6 \left( E_g + \frac{2}{3} \Delta \right)}
\]

Alternatively, an energy parameter \( E_p \) can be defined as:

\[
E_p = \frac{2m_0}{\hbar^2} P^2, \quad \text{so that} \quad M_b = \frac{m_0}{6} E_p
\]

Ref: Chuang
The other polarizations, spin, and LH band

Same result $M_b^2$ is obtained for

* For $\hat{e} = \hat{y}$ or $\hat{e} = \hat{z}$ (cubic symmetry)

* For the other spin component of the CB, $\langle iS \downarrow |$

* For the transition between the LH band (per spin),

$$\left| \langle iS \downarrow | ex\frac{3}{2},\frac{1}{2} \rangle \right|^2 + \left| \langle iS \downarrow | ex\frac{3}{2},-\frac{1}{2} \rangle \right|^2$$
Joint Density of States (also called reduced DOS)

This is an important piece that appears inside total transition rate expressions.

Single Parabolic Band DOS:

\[ N_m(E) = \sum_{\vec{k} \in \text{BZ}} \sum_{\sigma} \delta(E - E_m(\vec{k})) \]

For a parabolic band: \( E - E_c = \frac{\hbar^2 k^2}{2m_{\text{dos}}} \)

\[ N_m(E) = \sqrt{2} \left( m_{\text{dos}}^* \right)^{3/2} \frac{\sqrt{E - E_c}}{\pi^2 \hbar^3} \]

Joint DOS of CB-VB:

\[ N_{cv}(\hbar \omega) = \sum_{\vec{k} \in \text{BZ}} \sum_{\sigma} \delta(E_{\sigma}(\vec{k}) - E_c(\vec{k}) + \hbar \omega) \]

\[ N_{cv}(\hbar \omega) = \sqrt{2} \left( m_r^* \right)^{3/2} \frac{\sqrt{\hbar \omega - E_g}}{\pi^2 \hbar^3} \]
Absorption Rate (Final Expression)

With all these ingredients the bulk absorption rate for unpolarized light becomes:

$$W_{abs} = \frac{\pi e^2 \hbar n_{ph}}{m_0^2 \hbar \omega \varepsilon} \left(2 M_b^2\right) N_{cv}(\hbar \omega)$$

where $N_{cv}(\hbar \omega)$ is the joint density of states (JDOS) given by:

$$N_{cv}(\hbar \omega) = \sqrt{2} \frac{(m_r^*)^{3/2}}{\pi^2 \hbar^3} \sqrt{\hbar \omega - E_g}$$
Radiative e-h Recombination Time: Emission

In the case of interband recombination rate of an e with a hole at the same \( k \) state, we integrate over all possible photon states

\[
W_{em} = \frac{\pi e^2 \hbar}{m^2_0 \hbar \omega \epsilon} \left( n_{ph} + 1 \right) |\mathbf{a} \cdot \mathbf{p}_{if}|^2 \rho_a(\hbar \omega)
\]

\[
\rho(\hbar \omega) = \frac{2\omega^2}{2\pi^2 \hbar^2 \nu^3}
\]

3D total photon DOS

For \( n_{ph} = 0 \), \( W_{em} \rightarrow W_{spon} \)

Associated e-h radiative recombination time is

\[
\tau_0 = \frac{1}{W_{spon}}
\]
Interband Transitions in Quantum Wells

transitions between subbands derived from different bulk bands

Subband Wavefunctions

\[ \psi_c^n = \frac{1}{\sqrt{AW}} e^{ik_c \cdot \rho} g_c^n(z) u_c^n k_c \]

\[ \psi_v^m = \frac{1}{\sqrt{AW}} e^{ik_h \cdot \rho} \sum_v g_v^m(z) u_v^m k_h \]

Normalization area
Well width
Due to mixing in the VB

3D to 2D: Optical transitions are affected in two ways

- Form of JDOS
- Momentum matrix element; anisotropy is now genuine

Ref: Singh
Momentum Matrix Element in QWs

In going from 3D to 2D:

\[ p_{if}^{3D} = \frac{1}{V} \int e^{i(k_e - k_h) \cdot r} \langle u_v^\nu | p_a | u_c \rangle \, d^3r \]
\[ \rightarrow p_{if}^{2D} = \frac{1}{AW} \sum_{n} \langle g_v^{\nu m} | g_n^n \rangle \int e^{i(k_e - k_h) \cdot \rho} \langle u_v^{\nu m} | p_a | u_c \rangle \, d^2\rho \]

env. fn. overlap along growth dir.
in-plane overlap

[ Other term, \( p_a \) acting on \( g_c^n(z) \) leaves \( \langle u_v^{\nu m} | u_c \rangle = 0 \) at the same \( \vec{k} \) state]

Unlike 3D, polarization dependence exists in 2D

Notation

\[ \begin{align*}
\text{TE (to growth axis): Electric field in QW plane} \\
\text{TM (to growth axis): Electric field along growth axis}
\end{align*} \]

Ref: Singh
Let the QW growth axis be $z$ axis

**TE** (Optical electric field in $xy$ plane)

Optical dipole matrix element is averaged over the azimuthal angle

From both **HH** bands to $\langle iS \uparrow' \rangle$

$$|\hat{e} \cdot \mathbf{p}_{cv}|^2 \equiv \left| \langle \hat{e} \cdot \mathbf{M}_{c-hh} \rangle \right|^2 = \frac{1}{2\pi} \int_0^{2\pi} d\phi \left| \hat{e} \cdot \mathbf{M}_{c-hh} \right|^2$$

$$= \frac{1}{2\pi} \int_0^{2\pi} d\phi \left( \cos^2 \theta \cos^2 \phi + \sin^2 \phi \right) \frac{P_x^2}{2}$$

$$= \frac{3}{4} \left( 1 + \cos^2 \theta \right) M_b^2$$

From both **LH** bands to $\langle iS \downarrow' \rangle$

$$\langle |\hat{e} \cdot \mathbf{M}_{c-hh}|^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi \left( \left| \left\langle iS \downarrow | \mathbf{p}_x | \frac{3}{2}, \frac{1}{2} \right\rangle \right|^2 + \left| \left\langle iS \downarrow | \mathbf{p}_x | \frac{3}{2}, -\frac{1}{2} \right\rangle \right|^2 \right)$$

$$= \left( \frac{2}{3} \sin^2 \theta \langle \cos^2 \phi \rangle + \frac{1}{6} \cos^2 \theta \langle \cos^2 \phi \rangle + \frac{1}{6} \langle \sin^2 \phi \rangle \right) P_x^2$$

$$= \left[ \sin^2 \theta + \frac{1}{4} \left( \cos^2 \theta + 1 \right) \right] M_b^2$$

$$= \left( \frac{5}{4} - \frac{3}{4} \cos^2 \theta \right) M_b^2$$

Same results for the other CB spins not considered

Ref: Chuang
\( \langle \hat{e} \cdot M_{c-lh} \rangle \) = \( \frac{1}{2\pi} \int_0^{2\pi} d\phi |\hat{e} \cdot M_{c-lh}|^2 = \frac{3}{2} \sin^2 \theta M_b^2 \)

\( \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi \left( \left| \langle iS \downarrow |ez| \frac{3}{2}, \frac{1}{2} \rangle \right|^2 + \left| \langle iS \downarrow |ez| \frac{3}{2}, -\frac{1}{2} \rangle \right|^2 \right) \)

\[ = \left( \frac{1}{6} \sin^2 \theta + \frac{2}{3} \cos^2 \theta \right) P_x^2 \]

\[ = \frac{1 + 3 \cos^2 \theta}{2} M_b^2 \]

**Table 9.1 Summary of the Momentum Matrix Elements in Parabolic Band Model \(|\hat{e} \cdot p_{ce}|^2 = |\hat{e} \cdot M|^2\)**

| Bulk \(|\hat{x} \cdot p_{ce}|^2 = |\hat{y} \cdot p_{ce}|^2 = |\hat{z} \cdot p_{ce}|^2 = M_b^2 = \frac{m_0}{6} E_p \)| | Quantum Well |
|---|---|
| **TE Polarization \((\hat{e} = \hat{x} \text{ or } \hat{y})\)** | **TM Polarization \((\hat{e} = \hat{z})\)** |
| \( \langle |\hat{e} \cdot M_{c-hh}|^2 \rangle = \frac{3}{4} (1 + \cos^2 \theta) M_b^2 \) | \( \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle = \frac{3}{4} \sin^2 \theta M_b^2 \) |
| \( \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle = (\frac{3}{4} - \frac{3}{4} \cos^2 \theta) M_b^2 \) | \( \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle = \frac{1}{2} (1 + 3 \cos^2 \theta) M_b^2 \) |

**Conservation Rule**

\( \langle |\hat{e} \cdot M_{c-h}|^2 \rangle + \langle |\hat{y} \cdot M_{c-h}|^2 \rangle + \langle |\hat{z} \cdot M_{c-h}|^2 \rangle = 3M_b^2, \ (h = hh \text{ or } lh) \)

\( \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle + \langle |\hat{e} \cdot M_{c-lh}|^2 \rangle = 2M_b^2 \)

Ref: Chuang
Back to Absorption Rate in QWs

JDOS in 2D:

\[
\frac{N_{c2D}^{2D}(\hbar \omega)}{W} = \frac{m^*}{\pi \hbar^2 W} \sum_{nm} \langle g^{nm}_v | g^n_c \rangle \theta(E_{nm} - \hbar \omega)
\]

\[
E_{nm} = E_{gap} + E^m_c + E^m_v
\]

\[
\alpha(\hbar \omega) = \frac{\pi e^2 \hbar}{m^* \epsilon_0} \frac{1}{(\hbar \omega)} |a \cdot \mathbf{p}_{if}|^2 \frac{N_{2D}(\hbar \omega)}{W} \sum_{nm} f_{nm} \theta(E_{nm} - \hbar \omega)
\]

\[
f_{nm} = \left| \sum_\nu \langle g^{\nu m}_v | g^n_c \rangle \right|^2
\]

Observe that even-odd parity transitions are not allowed due to vanishing of this overlap.

Figure 9.7: Calculated absorption coefficient in a 100 Å GaAs/Al0.4Ga0.6As quantum well structure for in-plane polarized light. The HH transition is about three times stronger than the LH transition in this polarization. In a real material excitonic transition dominate near the bandedges as discussed in the next chapter.

Ref: Singh
Indirect Interband Transitions in Bulk

Common Indirect Se/c: Si, Ge, C, AlAs, GaP, AlP, SiC, AlN (zb)

No energy cons.

2nd Order Matrix Element:

\[ W_{if} = \frac{2\pi}{\hbar} \int \left| \sum \frac{\langle f| H_{\text{per}} |n\rangle \langle n| H_{\text{per}} |i\rangle}{E_i - E_n} \right|^2 \delta(E_f - E_i) \frac{d^3k}{(2\pi)^3} \]

\[ H_{\text{per}} = H_{\text{ph}} + H_{\text{ep}} \]

Weight of these two pathways \( \propto \frac{1}{|E_i - E_n|^2} \)

With photon energies smaller than the direct band gap, intermediate transitions can occur since energy need not be conserved.

Ref: Singh
Pathways which require phonon emission/absorption

Form of the matrix elements:

\[
W_{ij}(k) = \frac{2\pi}{\hbar} \int \delta(E_f - E_i) \frac{d^3k}{(2\pi)^3} \left\{ |M_{em}|^2 + |M_{abs}|^2 \right\}
\]

\[
M_{abs} = \left| \frac{\langle c, k + q | H_{ep} | c, k \rangle^2 \langle c, k | H_{ph} | v, k \rangle^2}{(E_{\Gamma \Gamma} - \hbar \omega)^2} \right|
\]

\[
M_{em} = \left| \frac{\langle c, k - q | H_{ep} | c, k \rangle^2 \langle c, k | H_{ph} | v, k \rangle^2}{(E_{\Gamma \Gamma} - \hbar \omega)^2} \right|
\]

direct optical transitions

e-phonon scattering matrix elements
due to optical phonon intervalley scattering
with the associated matrix element:

\[
M_q^2 = \frac{\hbar D_{ij}^2}{2\rho V \omega_{ij}} \left\{ \frac{n(\omega_{ij})}{n(\omega_{ij}) + 1} \right\}
\]

\[D_{ij}: \text{Deformation potential}\]
\[\rho: \text{Mass density}\]
\[\omega_{ij}: \text{Intervalley phonon frequency}\]
\[n(\omega_{ij}): \text{phonon occupancy (BE distr.)}\]

Ref: Singh
For parabolic bands, the absorption rate results in:

\[
W_{\text{abs}}(\hbar \omega) = \frac{M^2_{\text{ph}} D^2_{ij} J_v (m_v m_v)^{3/2}}{8 \pi^2 (E_g \Gamma - \hbar \omega)^2 \hbar^6 \rho \omega_{ij}} \times \left[ n(\omega_{ij}) \left( \hbar \omega - E_{gk'} + \hbar \omega_{ij} \right)^2 + \{n(\omega_{ij}) + 1\} \left( \hbar \omega - E_{gk'} - \hbar \omega_{ij} \right)^2 \right]
\]

Photon-related matrix element

\[
M^2_{\text{ph}} = \frac{e^2 \hbar n_{\text{ph}} |a \cdot p_{\text{if}}|^2}{2m^2_0 \epsilon \omega}
\]

Note the contrast in \(W_{\text{abs}}\)

Direct Bandgap: \(\left( \hbar \omega - E_g \right)^{1/2}\)

Indirect Bandgap: \(\left( \hbar \omega - E_{\text{th}} \right)^2\)

Figure 9.10: Absorption coefficient of Si and Ge. Also shown is absorption coefficient for amorphous silicon which is almost like a direct gap semiconductor, since \(k\)-selection is not applicable.

In amorphous se/c, \(k\)-conservation requirement is relaxed (no periodicity, xtal momentum not a good quantum label) This results in higher absorption coefficient

Ref: Singh
**Intraband Transitions in Bulk Se/c**

- As each band at a $k$-state is single-valued 1\textsuperscript{st} order vertical intraband transitions are not possible.

- Intraband transitions must involve some second mechanism (phonon, ionized imp, defects...) to ensure momentum conservation.

- Intraband transitions are also known as **free carrier absorption** and are effective in the cladding layers of lasers.
Drude Model (to explain free carrier absorption)

\[ m^* \ddot{x} + m^* \gamma \dot{x} + m^* \omega_0^2 = eE_0 \cos(\omega t) \]

w/o scattering no net energy xfer; e’s oscillate back and forth within the band

By introducing a scattering mechanism, energy gained by the e in one cycle will be partially lost in the form of, say phonon emission by the electron.

\[ \alpha(\hbar \omega) \propto \frac{1}{\omega^2} \]

\[ \propto \frac{1}{\mu} \text{ mobility} \]

If the mobility is large (weak scattering) absorption coefficient becomes small
Intraband Transitions in Quantum Wells

Since a number of subbands may originate from the same bulk band, certain inter-subband transitions (CB1-CB2) may be termed as intraband transitions in QWs.

Such inter-subband transitions have great importance for far infrared detectors and forms the basis of Quantum Cascade Lasers.

\[
\psi_1(k, z) = g_1(z) e^{ik \cdot \rho} u_{nk}^1(r) \\
\psi_2(k, z) = g_2(z) e^{ik \cdot \rho} u_{nk}^2(r)
\]

orthogonal

Approximately same for the CB

Ref: Singh
Momentum Matrix Element:

\[
\mathbf{p}_{if} = -\frac{i\hbar}{W} \int g^2(z) e^{-i\mathbf{k} \cdot \rho} \mathbf{a} \cdot \nabla g^1(z) e^{i\mathbf{k} \cdot \rho} d^2\rho \, dz
\]

If the polarization lies on the QW plane, then due to the orthogonality of the remaining envelope parts \((g^1, g^2)\), \(p_{if} = 0\)

Thus for EM wave polarized in the plane of the QW, inter-subband transition rate is zero (This can be relaxed under strong mixing of the cell-periodic parts as in the VB.)

For EM wave polarized along the QW growth axis (say \(z\)), we get

\[
\mathbf{p}_{if} = -\frac{i\hbar}{W} \int g^2(z) \hat{z} \frac{\partial}{\partial z} g^1(z) \, dz
\]

\(|\mathbf{p}_{if}| \approx \frac{\hbar}{W}\)

Brings \(g^1\) to the same parity with \(g^2\)

Ref: Singh