Bilkent University, Department of Physics

PHYS 453: Nuclear & Particle Physics

Second Homework

Due Date: 9 March 2012

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1. The Cartesian components of the spin operator for spin-1 are given as:

$$S_x = \hbar \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \; S_y = \hbar \begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix}, \; S_z = \hbar \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Determine the corresponding eigenvalues and eigenvectors of each spin component.

2. For two particles of spin-2 and spin-3/2, if their orbital angular momenta are zero, and the total spin of the composite system is 5/2, with its z-component being –1/2, then what values are possible for a measurement of S_z on the spin-2 particle? What is the probability of each?

3. For the Pauli spin matrices: (here, I is a 2×2 identity matrix)
   a) Show that $\sigma_i \sigma_j = I\delta_{ij} + i\epsilon_{ijk}\sigma_k$, where ϵ_{ijk} is the Levi-Civita symbol,
   b) using part (a) show that the commutator: $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$,
   c) show the anticommutator: $\{\sigma_i, \sigma_j\} = 2I\delta_{ij}$,
   d) for any two vectors $\vec{a}$ and $\vec{b}$, show that $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) = (\vec{a} \cdot \vec{b})I + i\vec{\sigma} \cdot (\vec{a} \times \vec{b})$.

4. Nuclear physicists traditionally work with 'half-life' ($t_{1/2}$) instead of the mean lifetime (τ); $t_{1/2}$ is the time it takes for half of the members of a large sample to decay. For exponential decay, derive the formula for $t_{1/2}$ as multiple of τ.
