1) (25 points) Multi-electron \( g \)-factor
For the state with \( LS \) term symbol \(^3P_2\), calculate its \( g \)-factor and sketch the shifts of all magnetic sublevels as a function of external magnetic field, \( B \).

2) (35 points) Three-Level Rate Equations
Erbium-doped fiber amplifier operating at the 1.55 \( \mu \)m wavelength is based on the atomic transitions within the three levels (shown on the right) of the \( \text{Er}^{3+} \) ion embedded, say in silica fiber. Resonant with the \(|1\rangle \leftrightarrow |3\rangle\) transition, a pump \( I_p \) is present, and the main (laser) signal \( I_s \) originates from the \(|2\rangle \leftrightarrow |1\rangle\) transition. Assume that a non-radiative decay governed by the rate \( \Gamma_{NR} \) is also present as shown on the right. To study the population dynamics, using Einstein \( A \) and \( B \) coefficients (assumed to be state-independent) accompanying relevant transitions:

(a) Obtain the population rate equations for all three levels, \( n_1(t), n_2(t), n_3(t) \), with \( n_1(t) + n_2(t) + n_3(t) = 1 \), i.e., normalized to total population.

(b) Next, assuming that the non-radiative rate is much faster, redo part (a) for \( n_3(t) \sim 0 \).

(c) Obtain the expression for the steady-state value of \( n_2 \) for part (b).

(d) The \( LS \) term symbols are also specified in the Figure. Do you realize something bizarre regarding optical transitions? (+10 points bonus, if you also have an explanation for it)

3) (40 points) Two-Level Rabi Oscillations
A two-level system having an energy separation \( \hbar \omega_0 = E_e - E_g \) is illuminated with a (classical) electric field \( F_0 \cos \omega t \), with a detuning \( \Delta \equiv \omega_0 - \omega \). We can write the wave function of the atom as \( |\psi(t)\rangle = C_g(t)e^{-iE_g t/\hbar}|g\rangle + C_e(t)e^{-iE_e t/\hbar}|e\rangle \). Within the electric dipole coupling described by the parameter \( V \equiv -\langle e|\vec{d}|g\rangle \cdot \vec{F}_0 \) (you can take \( V^* = V \)), generalized Rabi frequency \( \Omega_R \equiv \sqrt{\Delta^2 + V^2/\hbar^2} \), and neglecting the spontaneous emission term:

(a) Obtain the coupled differential equations satisfied by \( C_g(t), C_e(t) \).

(b) Next, applying the so-called rotating wave approximation (RWA) which amounts to discarding the rapidly oscillating terms \( e^{\pm i(\omega + \omega_0)t} \), simplify your expressions in (a).

(c) Solve for \( C_e(t) \) within RWA subject to \( C_g(0) = 1, C_e(0) = 0 \).