1) (30 points) Relativistic Corrections
Find the first-order relativistic corrections due to (i) spin-orbit interaction, (ii) relativistic mass correction, and (iii) Darwin terms, for the 2s Hydrogen state, given by
\[ \varphi(r) = \frac{\pi}{(2\pi a_B)^{3/2}} \left( 1 - \frac{r}{2a_B} \right) e^{-r/2a_B}. \]
Put your final expressions in the form, \( m_e c^2 \alpha^n \).

2) (20 points) Hyperfine Interaction
Determine the hyperfine shift of the 1s state for deuterium atom (an isotope of hydrogen) which has nuclear spin-1.

3) (25 points) Exchange Energy
For two electrons having spatial wave functions \( \varphi_a(\vec{r}) \) and \( \varphi_b(\vec{r}) \), give the explicit expression for their exchange energy \( K_{ab} \). Using this expression prove that always \( K_{ab} > 0 \).
*Hint:* One way is to make use of Fourier transform properties.

4) (25 points) Spectroscopic Notation and Hund’s Rules
Find all possible terms in LS notation for oxygen \((1s^22s^22p^4)\) and fluorine \((1s^22s^22p^5)\). For either case identify the term for the ground state.

---

**Formula Reminder:**
\[
H_{SO} = \lambda \vec{l} \cdot \vec{s}; \quad H_R = -\frac{p^2}{8m_e^2c^2}; \quad H_D = \frac{e^2 \hbar^2}{8m_e^2c^2\epsilon_0} \delta(\vec{r})
\]
\[
H_{hf} = A \left[ \frac{3}{8\pi \epsilon_0} \left( \vec{l} \cdot \vec{I} \right) + \frac{3}{8\pi \epsilon_0} \left( \frac{3(\vec{s} \cdot \vec{r})(\vec{l} \cdot \vec{r})}{r^2} - (\vec{s} \cdot \vec{I}) \right) + \left( \vec{s} \cdot \vec{I} \right) \delta(\vec{r}) \right]
\]
\[
\langle \frac{1}{r} \rangle_{nl} = \frac{Z}{a_B n^2}; \quad \langle \frac{1}{r^2} \rangle_{nl} = \frac{Z^2}{a_B^2 n^4 (l+1/2)}; \quad \langle \frac{1}{r^3} \rangle_{nl} = \frac{Z^3}{a_B^3 n^6 (l+1)(l+1/2)}
\]
\[ a_B = \frac{\hbar}{m_e c}; \quad \frac{e^2}{4\pi \epsilon_0} = \alpha \hbar c \]