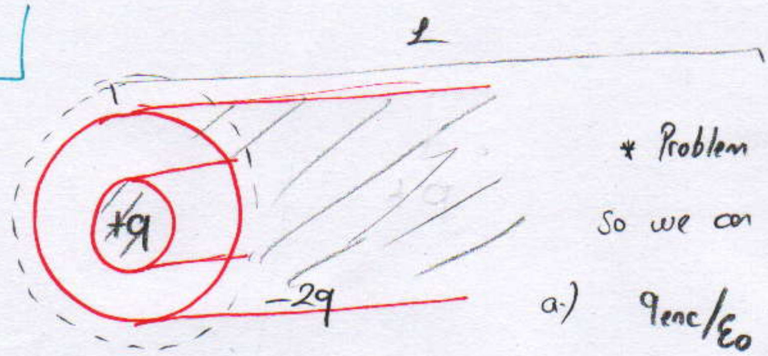
Solutions To HW III

Ch 27 Pr 5



* Problem has spherical symmetry:

So we can write Gauss Law as follows:

a-) $q_{enc}/\varepsilon_0 = \oint \vec{E}\cdot d\vec{A} = \int E\,dA$

$= E \int dA = 2\pi r L E$

$E = \dfrac{q_{enc}}{2\pi\varepsilon_0} \cdot \dfrac{1}{rL}$ here $q_{enc} = +q-2q$
$= -q$

$$\boxed{E = -\frac{q}{2\pi\varepsilon_0 r L}}$$

(−) sign indicates E fild is radially inward.

b-) Inside the conducting shell there exists no electric field. So $q_{enc}=0$.
This means -q is at outer side of shell, -q inner side of shell.

c-) Again applying Gauss Law;

$q_{enc} = +q$

$$q/\varepsilon_0 = \oint \vec{E}\,d\vec{A} = E\,2\pi r L$$

$$\boxed{E = q/2\pi\varepsilon_0 r L}$$

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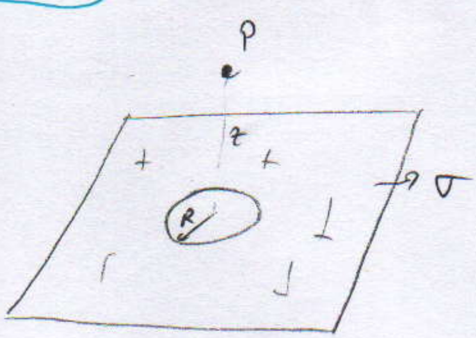

⇒ As we know for an infinite sheet of charge $\vec{E}$ field is approximated by:

$$E_z = \frac{\sigma}{2\varepsilon_0}$$ (Page 594 of your book)

⇒ So if we simply extract a uniform disk of charge from above equation;

$$E_{Tot} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$E_{Tot} = \frac{\sigma}{2\varepsilon_0}\frac{z}{\sqrt{z^2+R^2}}$$

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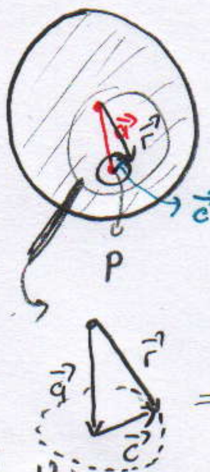

$\Rightarrow \vec{r} - \vec{c} = \vec{a}$

a-)

$$q_{enc}/\varepsilon_0 = \oint \vec{E} d\vec{A} = 4\pi r^2 E$$

charge is given by $\Rightarrow 4\pi r^2 \rho dr$

so;

$$q_{enc} = \int_0^r 4\pi \rho r^2 dr = 4\pi \rho r^3/3$$

$$\boxed{E = \rho r/3\varepsilon_0}$$

b-) Using superposition principles:

$\vec{E_h} = \vec{E} - \vec{E_c}$ where $\vec{E_h}$ is the field in hole, $\vec{E}$ is the field for full sphere and $\vec{E_c}$ is the field generated by the matter that would have been in the hole.

We can simply write $\vec{E_c} = \rho\vec{c}/3\varepsilon_0$ from part a.

Also;

$\vec{E} = \rho\vec{r}/3\varepsilon_0$ again from part a:

$$\frac{\rho}{3\varepsilon_0}\underbrace{(\vec{r} - \vec{c})}_{\vec{a}} = \boxed{\vec{E}_h = \frac{\rho}{3\varepsilon_0}\vec{a}}$$

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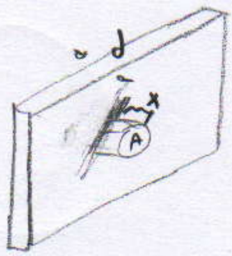

a-) for, inside of slab: $q_{enc} = \rho A x$ simply.

Then

$$\underbrace{\varepsilon_0 \oint \vec{E}\, d\vec{A}}_{2EA} = \rho A x \qquad E = \frac{\rho x}{2\varepsilon_0}$$

Since it is from median plane to a surface, and by symmetry $E=0$ along the median plane:

$$\boxed{E = \frac{\rho x}{\varepsilon_0}}$$

b-) Outside the slab; between median plane to a surface ~~a~~ distance x

$q_{enc} = \rho A\, d/2$, not depend on x since charge exists only at $d/2$ length.

$$E = \frac{\rho A d/2}{\varepsilon_0 A} = \boxed{\frac{\rho d}{2\varepsilon_0} = E}$$

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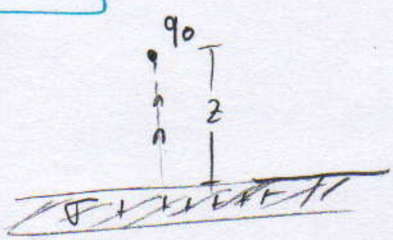

a-) Simply $W = -\int F dz$ (−) sign comes because here negative work has been done.

$$W = -Fz = -E q_0 z$$

$$\boxed{W = -\frac{\sigma}{2\varepsilon_0} q_0 z}$$

b-) we can write $W = q\Delta V$ $\quad -\frac{\sigma}{2\varepsilon_0} \cancel{q_0} z = \cancel{q_0} \Delta V \Rightarrow V - V_0 = -\frac{\sigma}{2\varepsilon_0} z$

$$\boxed{V = V_0 - \frac{\sigma}{2\varepsilon_0} z}$$

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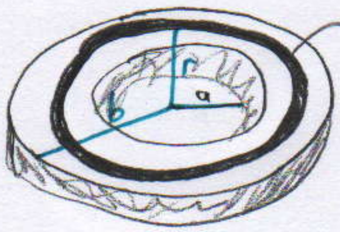
Pr 10



$dA = 2\pi r\,dr$

$$dq = \sigma\,dA = \frac{k}{r^3}\,2\pi r\,dr = \frac{2\pi k}{r^2}\,dr$$

$$dV = dq/4\pi\varepsilon_0 r$$

$$V = \int_a^b \frac{k}{2\varepsilon_0}\frac{dr}{r^3} = \frac{k}{4\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{b^2}\right) = \frac{k}{4\varepsilon_0}\,\frac{b^2-a^2}{b^2a^2}$$

$\Rightarrow$ Total charge on annulus is;

$$Q = \int_a^b \frac{2\pi k}{r^2}\,dr = 2\pi k\left(\frac{1}{a}-\frac{1}{b}\right) = 2\pi k\,\frac{b-a}{ba}$$

$$V = \frac{k}{4\varepsilon_0}\,\frac{(b+a)(b-a)}{ba.\,ba} \Rightarrow \frac{Q}{2\pi k} \qquad \boxed{V = \frac{Q}{8\pi\varepsilon_0}\,\frac{(b+a)}{ab}}$$

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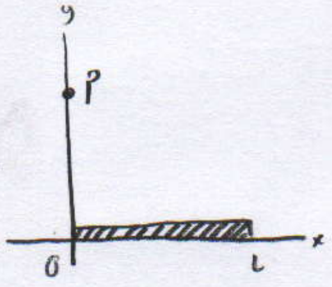
Pr 13



a-) From your book page 644, potential for a uniform line of charge is given by:

$$dV = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,dx}{\sqrt{x^2+y^2}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{kx\,dx}{\sqrt{x^2+y^2}} \qquad \lambda = kx$$

$du = 2x\,dx \;-\int \frac{du}{2\sqrt{u}}$

$$V = \frac{k}{4\pi\varepsilon_0}\sqrt{x^2+y^2}\,\Big|_0^L$$

$$\boxed{V = \frac{k}{4\pi\varepsilon_0}\left[\sqrt{L^2+y^2}-y\right]}$$

b-) $$E_y = -\frac{\partial V}{\partial y} = \boxed{\frac{k}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right)}$$
using V in part a

c-) Since we don't know x variation of V, we cannot find using part a.

d-) We want ratio $$\frac{1}{2} = \frac{\frac{k}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right)}{\frac{k}{4\pi\varepsilon_0}\,L}$$ $\Rightarrow V$ at $y=0$ $\Rightarrow L^2+y^2 = L^2/4$ $\quad \boxed{y = \frac{3}{4}L}$