MATH 132, Finite and Discrete Mathematics, Spring 2016

Course specification

Laurence Barker, Bilkent University, version: 4 March 2016.

Course Aims: To supply an introduction to some concepts and techniques associated with discrete methods in pure and applied mathematics; to supply an introduction or reintroduction to the art of very clear deductive explanation.

Course Description: The terms *combinatorics* and *discrete mathematics* have similar meanings. The former refers to an area of pure mathematics concerned with mathematical objects that do not have very much topological, geometric or algebraic structure. The latter refers to an area of applicable mathematics that rose to prominence with the advent of electronic computers and information technology. Of course, the two cultures overlap considerably and cannot be clearly distinguished from each other.

In the 1950s and 60s, pioneers of computing and computer science found that the established styles of applied mathematics were unsuitable for the new kinds of problem that were appearing. Unlike the classical applied fields such as differential calculus, linear algebra and statistics, the new kind of mathematics was not conducive to formalism, that is to say, methods of calculation based on manipulation of written symbols. Applied mathematicians found that they needed to adopt a conceptual approach which had previously been mostly confined to pure mathematics and some areas of physics.

In discrete mathematics, as opposed to classical applied mathematics, solutions to problems tend to comparatively unsystematic, though certain fundamental ideas do tend to be used quite frequently. For that reason, the study of discrete mathematics depends heavily on the art of *very clear deductive explanation*, which will be emphasized throughout the course.

The course is intended for students who have little or no previous experience of this kind of mathematics. There are no course prerequisites, in fact, proficiency at formal methods of symbolic manipulation will confer no advantage.

We shall be studying three main areas, separate but with some interactions: (1) graph theory; (2) relations and enumerative combinatorics; (3) coding theory.

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Assistant: Gökalp Alpan, Office SA-144, gokalp ta fen tod bilkent tod edu tod tr .

Text: R. P. Grimaldi, "Discrete and Combinatorial Mathematics", 5th Ed. (Pearson, 2004).
Some notes will be supplied, on my webpage, for some of the syllabus material.

Classroom location and schedule: All the classes are in room SAZ 04. The times are Wednesdays 09:40 - 10:30 and Fridays 10:40 - 12:30.

Office Hours: In room SA-129 (same building as the classroom), Wednesdays 08:40 - 09:30.

One purpose of Office Hours is to get help with the homework. Usually, homeworks are due in on a Friday. I suggest you do what you can of the homework and then come at ask me for help with the troublesome exercises on Wednesday morning. I will not solve the exercise for you, but I can give you some guidance on how to do it.

Office Hours is not just for the stronger students. If you cannot do the easy questions, and if you do not even understand very much of the course material then, (provided you have at least thought about it and have something to talk about), come and see me during office hours. If you think the best grade you can get is a C, then I will help you get that C. If you are hopelessly lost and heading directly for an F grade, I will not be annoyed about that, because I already know that there are always some students who are hopelessly lost; I cannot be of much help to them if they do not come to see me!

Class Announcements: All students, including any absentees from a class, will be deemed responsible for awareness of announcements made in class.

Revision Aid: Some past exams, with solutions, can be found in discretepastpapers.pdf, on my homepage.

Assessment

Homeworks: The only way to pick up skill at mathematics is through lots of practise.

You may not copy homeworks and you may not do paraphrase rewrites of homeworks by other people. If you break this rule, then you will not catch up in time for the Midterm and Final exams. Just as you cannot learn to swim by watching other people do it, you cannot aquire mathematical skill by just writing out arguments produced by others, not even if you feel that you are "understanding" it as you copy.

You should discuss the homework with each other. In this way, you will teach each other. If you cannot do a homework question, ask another student or ask me during office hours.

Participation: This will be a mark awarded to the whole section for collective academic behaviour and participation. Asking questions is usually very helpful. All communication should be addressed to the whole class. (Making a distracting noise by murmuring to your neighbour is not proper academic behaviour.)

Principle of marking: In mathematics, marks for written work are not awarded according to guesses about what the student might have had in mind. They are awarded according to *how helpful the written explanation would be to other students in the class.*

Grading percentages:
- Quizzes, participation and homework, 15%,
- Midterm 1, 25%, Friday, 11th March, 10:30 - 12:30.
- Midterm 2, 25%, (date to be arranged, possibly Friday 8th April, unconfirmed).
- Final, 35%.

Letter Grades: This is done by the "curve method". A grade C requires an understanding of the concepts and competence at the most routine exam questions. That fulfills the aim of the course: a competent grasp at an introductory level.

Some of the exercises and exam questions will be quite difficult. It has to be that way, not only for the benefit of the strongest students, but also because, without difficult questions, it would be hard to see the purpose of the art of *very clear deductive explanation.* However, students aiming for a grade C need not worry about being unable to do the more difficult questions.

FZ Grades: These will be awarded to students satisfying at least one of the following conditions: (a) Very poor attendance (less than 50% as measured by quizzes); or (b) very poor

Midterm marks (incompetence routine questions); or (c) poor attendance and poor Midterm marks.

Attendance: A minimum of 75% attendance is compulsory (because of the inaccuracy of the Quiz sampling method, absence from 50% of the quizzes will be deemed a failure of attendence).

Syllabus

The topics in square brackets, below, are not in the textbook and are not on the examinable syllabus. The format for each item is: week number; Monday date; topics; textbook section numbers.

1: 25 Jan: Outline of course. Examples of problems in discrete mathematics.

2: 1 Feb: Further exercises in discrete mathematics. Argument by contradiction.

3: 8 Feb: Argument by minimal counter-example. The Principle of Mathematical Induction. Argument by mathematical induction, 4.1.

4: 15 Feb: Recursive definitions and various illustrations of mathematical induction, 4.2. Second order recurrence relations as an application of induction, 10.2.

5: 22 Feb: Graphs, sum of degrees formula. Circuits and Trees. 11.1, 11.2, 12.1. Multigraphs and directed multigraphs. Criteria for existence of Euler paths or Euler circuits, proved by mathematical induction, 11.3.

6: 29 Feb: Euler's characteristic formula for planar graphs, proved by mathematical induction. Techniques for proving non-planarity and, in particular, the non-planarity of the graphs K_5 and $K_{3,3}$, 11.4. [Discussion of Four-Colour Map Theorem and proof of a version with 5 colours.]

7: 7 Mar: Revision for Midterm 1, then Midterm 1 on Friday 11 March.

8: 14 Mar: Permutations, combinations, the Binomial theorem, 1.2, 1.3, 1.4.

9: 21 Mar: Sets and correspondences. Functions. Injections, surjections and bijections, 5.1, 5.2, 5.3, 5.6.

10: 28 Mar: Relations. Incidence matrices. Reflexive, irreflexive, symmetric, antisymmetric and transitive relations. Enumeration of relations using incidence matrices, 7.1, 7.2. Partial ordering relations, Hasse diagrams, [Dilworth's Theorem], 7.3.

11: 4 Apr: Revision for Midterm 2. (Date of Midterm 2 not yet confirmed.)

12: 11 Apr: Equivalence relations, 7.4. Stirling numbers of the second kind and enumeration of equivalence relations, 5.3. Inclusion-Exclusion Principle 8.1. [Proof of formula for Stirling numbers using Inclusion-Exclusion Principle.]

13: 18 Apr: Coding theory, Hamming metric, 16.5, 16.6. Hamming bound and Gilbert bound, 16.8.

14: 25 Apr: Parity-check and generator matrics, decoding using syndromes and coset leaders, 16.7, 16.8.

15: 2 May: [Proof of optimality of the decoding method.] Exercises in coding theory.

16: 21 Dec: Exercises in all topics and revision for Final Exam.

Midterm 1 Syllabus

The numberings are chapter and section numbers in Grimaldi.

• Mathematical Induction, 4.1, 4.2.

Test: Do you know what the term *inductive assumption* means? When writing out induction arguments, can you state the inductive assumption? (If not, ask me. Many people find this difficult.)

• Introduction to graph theory, 11.1.

Major result: Letting e be the number of edges, then $2e$ is equal to the sum of the degrees.

• Trees, 12.1.

Definition: A tree is a connected graph with no cycles.
Major result: Given a tree with n vertices and e edges, then $e = n - 1$.

• Euler paths, 11.3.

Euler's Path Theorem: Let r be the number of odd-degree vertices of a connected graph G. Then G has an Euler path if and only if $r = 0$ or $r = 2$. Also, G has an Euler circuit if and only if $r = 0$.

Test: Do you know how to find Euler paths for given graphs?

• Planar graphs, 11.4.

Euler's Characteristic Theorem: Given a connected planar graph G with n vertices and e edges, supposing some planar diagram of G has f faces, then $n - e + f = 2$.

Corollary: Given a connected graph G that is not a tree, and an integer c with $c \geq 3$, supposing that every cycle in G has length at least c, then $e \leq c(n-2)/(c-2)$.

• Second order recurrence relations, 10.1, 10.2.

Assuming $a \neq 0$ and $c \neq 0$, then formula for the solutions to $ax_{n+2} + bx_{n+1} + c = 0$ depends on whether or not the quadratic equation $aX^2 + bX + c = 0$ has a repeated solution. If there are distinct solutions α and β, then there exist A and B such that, for all n, we have $x_n = A\alpha^n + B\beta^n$. If there exists a unique solution, then there exist C and D such that, for all n, we have $x_n = (C + Dn)\alpha^n$.

Midterm 2 Syllabus

• Binomial coefficients, 1.3 (see also 1.1, 1.2).

• Relations 5.1. Injective, surjective, bijective functions. Composition of functions. 5.2, 5.3, 5.6.

• Incidence matrices, especially when used for counting, 7.1, 7.2.

• Equivalence relations, 7.4.

• Graph isomorphism, 11.2.

(Partial ordering relations and Dilworth's Theorem are not on the Midterm 2 syllabus but will appear on the Final syllabus. Embedding of graphs on real 2-manifolds such as the torus, Klein bottle and real projective plane are not on the syllabi of any of the exams.)

Final Syllabus

In the final, all topics on the course are examinable, but the focus will be on the following topics because we studied them after the Midterm 2 exam. The section numbers are from the Grimaldi textbook.

- Stirling numbers 5.3. (Counting arrangements of coloured balls in plain boxes, counting surjections, counting equivalence relations.)

- Coding theory.
  - Binary codes, Hamming metric 16.5, 16.6. (Highlight: Hamming bound and Gilbert bound in Exercise 16.7.12.)
  - Linear codes, decoding table, efficient decoding using syndromes, 16.7, 16.8.

Special Office Hours for Final Preparation: Wednesday 4 May, 08:40 - 10:30, initially in my Office SA-129, migrating to classroom SA-Z04 when my office becomes too crowded.

Midterm 2 Makeup: Thursday, 28 April, 17:30 - 19:30, SA-Z18.