# Course specification

MATH 227, *Introduction to Linear Algebra*, Spring 2019

Laurence Barker, Bilkent University. Version: 5 March 2019.

**Course Aims:** To aquire practical knowledge and skill in some important techniques of linear algebra, and to aquire theoretical understanding and ability to critically assess methods and propositions in the area.

**Course Description:** This is an introductory course with an emphasis on methods of calculation, but with a theoretical grounding that is self-contained and complete. Some victory conditions, for the student, include: an understanding of the notion of a vector space as something more than just a system of coordinates; an ability to apply the method of diagonalization, a clear grasp of the theory behind that method.

**Course Requirements:** An official prerequisite in the University Catalogue is MATH 106 but, in practice, no knowledge of calculus is needed.

**Instructor:** Laurence Barker, Office SAZ 129, barker at fen dot bilkent dot edu dot tr.

**Textbook:** Howard Anton, Chris Rorres, "Elementary Linear Algebra", 11th edition, Wiley, 2011, 2015. ISBN: 978-1-118-67745-2.

Warning: The Main course text is not the book with the same authors, same title, same edition number, same publisher but with a different ISBN. The main course text has the text "International Student Version" on the front cover, whereas the other book does not. The two books have different exercises.

**Midterm 1 revision notes:** see the file on my homepage: midterm1revision227Spr19.pdf
**Notes on diagonalization:** see the file on my homepage: diagonalization.pdf .

**Classes:** Tuesdays 10:40 - 12:30, Fridays 09:40 - 10:30, room MA-301.

**Office Hours:** Fridays 08:40 - 09:30, in my office, room SA-129.

Office hours is for *all* the students on the course, not just the proficient. If you are having difficulty with the course, then it is best to come to see me for advice.

## Weekly Syllabus

The format below is, *Week number; Monday date; Subtopics and textbook section numbers.* The numbering $m.n$ indicates Chapter $m$ Section $n$ in the Anton–Rorres textbook.

**1: 4 Feb:** Systems of linear equations, 1.1. Sketch of Markov chain scenario and the method of diagonalization.

**2: 11 Feb:** Gaussian and Gauss–Jordan elimination, 1.2. Matrics, 1.3.

**3: 18 Feb:** Elementary matrices, inversion of matrices by row operations 1.4, 1.5. Existence and uniqueness of solutions, 1.6.

**4: 25 Feb:** Determinants, their algebraic properties, their evaluation by row reduction and by cofactor expansion, 2.1, 2.2, 2.3.

**5: 4 Mar:** Euclidian spaces, norm, dot product, distance, angle, 3.1, 3.2.

**6: 11 Mar:** Pearson correlation coefficient (special notes). Introduction to Markov chains, 4.12.

**7: 18 Mar:** (No class on Friday.) Real vector spaces, subspaces, 4.1, 4.2.

**8: 25 Mar:** Linear independence, spanning, bases, dimension, 4.3, 4.4, 4.5.

**9: 1 Apr:** Linear transformations, inverses, composition, 8.1, 8.2, 8.3.

**10: 8 Apr:** Change of basis, 4.6. Row and column spaces, rank-nullity formula, 4.7, 4.8.

**11: 15 Apr:** Eigenvalues and eigenvectors, 5.1

**12: 22 Apr:** (No class on Tuesday.) Complex vector spaces, 5.3.

**13: 29 Apr:** Diagonalization, 5.2. Applications to Markov chains, 10.5.

**14: 6 May:** Inner product spaces, 6.1, 6.2.

**15: 13 May:** Gram–Schmidt orthogonalization, 6.3. Review for Final.

**Assessment:** The method of assessment is by curve. Grades F, FX, FZ are to be for candidates judged to be thoroughly incompetent in the routine course material.

- Quizzes and Participation 10%.
- Midterm I, 30%, Wednesday, 13 March, 18:00.
- Midterm II, 30%, Tuesday, 30 April, 18:00.
- Final, 30%, [date to be announced.]

FZ criteria: (1) less than 30% in sum of Midterm 1 and Midterm 2 marks, or (2) less than 50% in that sum and less than 75% attendance.

**Class Announcements:** All students, including any absentees from a class, will be deemed responsible for awareness of class announcements.

## Midterm 1 Exam Syllabus

**Solving linear equations,** $[\ |]$. Gaussian elimination, Gauss–Jordan elimination. 1.1, 1.2.

**Inverting matrices by Gauss–Jordan method,** $[\ |\ ]$. 1.3, 1.4, 1.5.

**Determinants and inverses by cofactor method,** $|\ \ |$. 1.6, 2.1, 2.2, 2.3 (but without Cramer's rule).

**Euclidian vector spaces,** $\|\,\|$. Vectors, 3.1. Norm, dot product, distance, Theorems 3.2.2, 3.2.4, 3.2.5. Orthogonality, Theorem 3.3.2. Pearson correlation coefficient (discussed in lectures and presented in notes on next page).

# Some notes on the Pearson correlation coefficient

The following theory, and some examples, are given in class on Tuesday 5 March. I supply this summary because the topic is not in the textbook.

Let $x_1$, ..., $x_n$ be sampled values of some variable, and let $y_1$, ..., $y_n$ be the corresponding sampled values of some other variable. The **Pearson correlation coefficient** $\rho$ of the two samples is calculated as follows:

**Step 1:** Calculate the average values

$$\overline{x} = (x_1 + ... + x_n)/n \;, \qquad\qquad \overline{y} = (y_1 + ... + y_n)/n \;.$$

**Step 2:** Calculate the centred vectors

$$\widetilde{x} = (\widetilde{x}_1, ..., \widetilde{x}_n) = (x_1 - \overline{x}, ..., x_n = \overline{x}) \;, \qquad\qquad \widetilde{y} = (\widetilde{y}_1, ..., \widetilde{y}_n) = (y_1 - \overline{y}, ..., y_n - \overline{y}) \;.$$

**Step 3:** Calculate $\rho = \dfrac{\widetilde{x}_\bullet \widetilde{y}}{\|\widetilde{x}\|\,\|\widetilde{y}\|}$ where

$$\widetilde{x}_\bullet \widetilde{y} = \widetilde{x}_1\widetilde{y}_1 + ... + \widetilde{x}_n\widetilde{y}_n \;, \qquad \|\widetilde{x}\| = \sqrt{\widetilde{x}_1^2 + ... + \widetilde{x}_n^2} \;, \qquad \|\widetilde{y}\| = \sqrt{\widetilde{y}_1^2 + ... + \widetilde{y}_n^2} \;.$$

Let us give two illustrative examples.

**Problem A:** Snails $A$, $B$, $C$ weigh 3, 3, 6 kilograms, respectively. The maximum speeds of $A$, $B$, $C$ are 3, 4, 8 meters per second, respectively. Let $\rho$ be the correlation coefficient for these samples of weights and speeds. Show that $\rho > 97/100$.

*Solution:* The given data is $(x_1, x_2, x_3) = (3, 3, 6)$ and $(y_1, y_2, y_3) = (3, 4, 8)$. The averages are $\overline{x} = (3 + 3 + 6)/3 = 4$ and $\overline{y} = (3 + 4 + 8)/3 = 5$. The centred vectors are $\widetilde{x} = (-1, -1, 2)$ and $\widetilde{y} = (-2, -1, 3)$. So $\|\widetilde{x}\|^2 = 1 + 1 + 4 = 6$ and $\|\widetilde{y}\|^2 = 4 + 1 + 9 = 14$. So

$$\rho = \frac{2 + 1 + 6}{\sqrt{6}\sqrt{14}} = \frac{9}{2\sqrt{21}} \;.$$

It follows that $\rho^2 = 81/84 > 96/100$, hence $\rho > 97/100$. $\square$

**2: Problem B.** Let $X$ be a variable with sample values 3, 3, 3, 1, 5 and let $Y$ be a variable with corresponding sample values 1, 3, 4, 3, 9. Calculate the Pearson correlation coefficient for the samples.

**Solution:** The coordinates of the vectors $x = (3, 3, 3, 1, 5)$ and $y = (1, 3, 4, 3, 9)$ have average values $\overline{x} = (3 + 3 + 3 + 1 + 5)/5 = 15/5 = 3$ and $\overline{y} = (1 + 3 + 4 + 3 + 9)/5 = 30/5 = 4$. The centred vectors are $\widetilde{x} = (0, 0, 0, -2, 2)$ and $\widetilde{y} = (-3, -1, 0, -1, 5)$. Therefore

$$\rho = \frac{\widetilde{x}.\widetilde{y}}{\|\widetilde{x}\| \,.\, \|\widetilde{y}\|} = \frac{0 + 0 + 0 + 2 + 10}{\sqrt{(0 + 0 + 0 + 4 + 4)(9 + 1 + 0 + 1 + 25)}} = \frac{12}{\sqrt{8 \times 36}} = 1/\sqrt{2} \;.$$