Course specification

MATH 227 Sections 1 and 2, *Introduction to Linear Algebra*, Fall 2018

Laurence Barker, Bilkent University. Version: 28 December 2018.

Course Aims: To aquire practical knowledge and skill in some important techniques of linear algebra, and to aquire theoretical understanding and ability to critically assess methods and propositions in the area.

Course Description: This is an introductory course with an emphasis on methods of calculation, but with a theoretical grounding that is self-contained and complete. Some victory conditions, for the student, include: an understanding of the notion of a vector space as something more than just a system of coordinates; an ability to apply the method of diagonalization, a clear grasp of the theory behind that method.

Course Requirements: An official prerequisite in the University Catalogue is MATH 106 but, in practice, no knowledge of calculus is needed.

Instructor: Laurence Barker, Office SAZ 129, barker at fen dot bilkent dot edu dot tr.

Assistant: Hatice Mutlu, hatice dot mutlu at bilkent dot edu dot tr.

Main course text: Howard Anton, Chris Rorres, "Elementary Linear Algebra", 11th edition, Wiley, 2011, 2015. ISBN: 978-1-118-67745-2.

Warning: The Main course text is not the book with the same authors, same title, same edition number, same publisher but with a different ISBN. The main course text has the text "International Student Version" on the front cover, whereas the other book does not. The two books have different exercises.

See STARS for some other recommended texts.

For notes on diagonalization, see the file on my homepage: diagonalization.pdf .

Classes: Section 1: Wednesdays 13:40 - 15:30, Fridays 15:40 - 16:30, room V 04. Section 2: Wednesdays 09:40 - 10:30, Fridays 10:40 - 12:30, room V 02.

Office Hours: Wednesdays 08:40 - 09:30, Fridays 16:40 - 17:30, in my office, room SA-129.

Office hours is for *all* the students on the course, not just the proficient. If you are having difficulty with the course, then it is best to come to see me for advice. You have nothing to lose by doing so, since otherwise I will anyway find out how bad you are when I mark the exams.

Weekly Syllabus

The format below is, *Week number; Monday date; Subtopics and textbook section numbers.* The numbering $m.n$ indicates Chapter m Section n in the Anton–Rorres textbook.

1: 24 Sep: Systems of linear equations, 1.1. Sketch of Markov chain scenario and the method of diagonalization.

2: 1 Oct: Gaussian and Gauss–Jordan elemination 1.2. Matrics 1.3.

3: 8 Oct: Elementary matrices, inversion of matrices by row operations 1.4, 1.5. Existence and uniqueness of solutions, 1.6.

4: 15 Oct: Determinants, their algebraic properties, their evaluation by row reduction and by cofactor expansion, 2.1, 2.2, 2.3.

5: 22 Oct: Euclidian spaces, norm, dot product, distance, angle, 3.1, 3.2.

6: 29 Oct: Pearson correlation coefficient (special notes). Markov chains, 10.5. Review for Midterm 1.

7: 5 Nov: Real vector spaces, subspaces, 4.1, 4.2. (Midterm 1 on 5 November at 18:00.)

8: 12 Nov: Linear independence, spanning, bases, dimension, 4.3, 4.4, 4.5.

9: 19 Nov: Linear transformations, inverses, composition, 8.1, 8.2, 8.3.

10: 26 Nov: Change of basis, 4.6. Markov chains, 4.12.

11: 3 Dec: Row and column spaces, Rank-Nullity Formula, 4.7, 4.8. (Midterm 2 on 3 December at 18:00.)

12: 10 Dec: Complex vector spaces, eigenvalues and eigenvectors.

13: 17 Dec: Diagonalization, 5.2. Applications to Markov chains, 10.5.

14: 24 Dec: Inner product spaces, Gram–Schmidt orthogonalization, 6.1, 6.2, 6.3.

For both sections, the last class is on Friday 28 December.

Assessment:

- Quizzes and Participation 10%.
- Midterm I, 30%, Monday, 5th November, 18:00.
- Midterm II, 30%, Monday, 3 December, 18:00.
- Final, 30%, Wednesday, 2 January, 12:30.

75% attendance is compulsory.

- Midterm II Makeup, Wednesday, 26 December, 18:00.

Class Announcements: All students, including any absentees from a class, will be deemed responsible for awareness of class announcements.

Midterm 1 Exam Syllabus

Solving linear equations, $[\ |]$. Gaussian elimination, Gauss–Jordan elimination. 1.1, 1.2.

Inverting matrices by Gauss–Jordan method, $[\ |\ ]$. 1.3, 1.4, 1.5.

Determinants and inverses by cofactor method, $|\ |$. 1.6, 2.1, 2.2, 2.3.

Geometry of $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^n$: $\|\|$. 3.1, 3.2, 3.3, 3.4.

Pearson correlation coefficient, $\bullet/\|\|\|\|$. (Discussed in lectures; also see notes below).

Midterm 2 Exam Syllabus

Real vector spaces, subspaces, 4.1, 4.2.

Linear independence, spanning, bases, dimension, 4.3, 4.4, 4.5.

Linear transformations, inverses, composition, 8.1, 8.2, 8.3.

Change of basis for vectors, 4.6.

Introduction to Markov chains, 4.12.

Final Exam Syllabus

4.6: Change of basis.
4.7 - 4.8: Rank-Nullity Formula,
5.1, 5.2: Eigenvalues, eigenvectors, diagonalization (including applications to Markov systems as discussed in lectures and the file diagonalization.pdf).
5.3: Complex vector spaces.
8.1: Linear maps.
8.3: Composition, inversion, matrix representation of linear maps.

Note: Due to disruption in week 14 due to heavy snow, material on inner product spaces in Chapter 6 is not on the syllabus for the Final Exam.

Some notes on the Pearson correlation coefficient

The following theory, and some examples, were given in class. I supply this summary because the topic is not in the textbook.

Let x_1, ..., x_n be sampled values of some variable, and let y_1, ..., y_n be the corresponding sampled values of some other variable. The **Pearson correlation coefficient** ρ of the two samples is calculated as follows:

Step 1: Calculate the average values

$$\overline{x} = (x_1 + ... + x_n)/n \ , \qquad \qquad \overline{y} = (y_1 + ... + y_n)/n \ .$$

Step 2: Calculate the centred vectors

$$\widetilde{x} = (\widetilde{x}_1, ..., \widetilde{x}_n) = (x_1 - \overline{x}, ..., x_n = \overline{x}) \ , \qquad \qquad \widetilde{y} = (\widetilde{y}_1, ..., \widetilde{y}_n) = (y_1 - \overline{y}, ..., y_n = \overline{y}) \ .$$

Step 3: Calculate $\rho = \dfrac{\widetilde{x}_\bullet \widetilde{y}}{\|\widetilde{x}\| \|\widetilde{y}\|}$ where

$$\widetilde{x}_\bullet \widetilde{y} = \widetilde{x}_1 \widetilde{y}_1 + ... + \widetilde{x}_n \widetilde{y}_n \ , \qquad \|\widetilde{x}\| = \sqrt{\widetilde{x}_1^2 + ... + \widetilde{x}_n^2} \ , \qquad \|\widetilde{y}\| = \sqrt{\widetilde{y}_1^2 + ... + \widetilde{y}_n^2} \ .$$

Let us give a small example problem. For another example, see Question 2 in the Practice Midterm 2.

Problem: Snails A, B, C weigh 3, 3, 6 kilograms, respectively. The maximum speeds of A, B, C are 3, 4, 8 meters per second, respectively. Let ρ be the correlation coefficient for these samples of weights and speeds. Show that $\rho > 97/100$.

Solution: The given data is $(x_1, x_2, x_3) = (3, 3, 6)$ and $(y_1, y_2, y_3) = (3, 4, 8)$. The averages are $\overline{x} = (3 + 3 + 6)/3 = 4$ and $\overline{y} = (3 + 4 + 8)/3 = 5$. The centred vectors are $\widetilde{x} = (-1, -1, 2)$ and $\widetilde{y} = (-2, -1, 3)$. So $\|\widetilde{x}\|^2 = 1 + 1 + 4 = 6$ and $\|\widetilde{y}\|^2 = 4 + 1 + 9 = 14$. So

$$\rho = \frac{2 + 1 + 6}{\sqrt{6}\sqrt{14}} = \frac{9}{2\sqrt{21}} \ .$$

It follows that $\rho^2 = 81/84 > 96/100$, hence $\rho > 97/100$. $\square$