MATH 132, Discrete and Combinatorial Mathematics, Spring 2018

Course specification

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version: 2 March 2018

Course Aims: To supply an introduction to some concepts and techniques associated with the discrete mathematical methods used in engineering and information technology; especially, to provide experience of the art of very clear deductive explanation.

Course Description: The terms *combinatorics* and *discrete mathematics* have similar meanings. The former refers to an area of pure mathematics concerned with mathematical objects that do not have very much topological, geometric or algebraic structure. The latter refers to an area of applicable mathematics that rose to prominence with the advent of electronic computers and information technology. The two areas overlap to the extent of often becoming indistinguishable.

In the 1950s and 60s, pioneers of computing and computer science found that the established styles of applied mathematics were unsuitable for the new kinds of problem that were appearing. Unlike the classical applied fields such as differential calculus, linear algebra and statistics, the new kind of mathematics was not conducive to formalism, that is to say, methods of calculation based on manipulation of written symbols. Applied mathematicians found that they needed to adopt a conceptual approach which had previously been confined mainly to pure mathematics and some areas of physics.

In discrete mathematics, as opposed to classical applied mathematics, solutions to problems tend to comparatively unsystematic, though certain fundamental ideas do tend to be used quite frequently. For that reason, the study of discrete mathematics depends heavily on the art of *very clear deductive explanation*, which will be emphasized throughout the course.

The course is intended for students who have little or no previous experience of this kind of mathematics. There are no course prerequisites, in fact, proficiency at formal methods of symbolic manipulation will confer no advantage.

We shall be studying three main areas, separate but with some interactions: enumerative combinatorics; relations and functions; graph theory.

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Text: R. P. Grimaldi, "Discrete and Combinatorial Mathematics", 5th Edition, New International Edition. (Pearson, 2014).

Some notes will be supplied, on my webpage, for some of the syllabus material.

Classroom hours: Tuesdays 15:40 - 16:30, Fridays 13:40 - 15:30, room G 236.

Office Hours: Tuesdays 16:40 - 17:30, room SAZ 129.

Office Hours is not just for the stronger students. If you cannot do the easy exercises, or if you do not even understand very much of the course material then, provided you do at least have some proactive questions, come and see me during office hours. Your ignorance and inability will not annoy me, since the experience of marking exams ensures that I am already aware of the miraculous extents that exist for those qualities.

Class Announcements: All students, including any absentees from a class, will be deemed responsible for awareness of announcements made in class.

Assessment:

- Quizzes, with lowest quiz mark dropped: 10%,
- Midterm 1: 30%, (Tuesday, 6th March, 20:00.)
- Midterm 2: 30%, (Tuesday, 10 April, 20:00.)
- Final: 30%.

Attendance: Quiz returns will be used to confirm attendance.

Letter Grades: This is done by the "curve method". High grades demand imagination and creativity. A grade C requires an understanding of the concepts and roughly correct solutions to most of the easier exam questions; that fulfills the minimal a competent grasp at an introductory level.

FZ Grades: These will be awarded to students satisfying one of the following conditions: (a) severely poor attendance as measured by quizzes; (b) severely poor midterm marks; (c) very poor attendance and very poor midterm marks.

Midterm 1 Exam Syllabus: The focus will be on the following sections of the textbook:

1.2: Permutations.
1.3: Binomial coefficients. Counting arrangements of plain balls in coloured boxes.
1.5: Catalan numbers.
4.1, 4.2: Mathematical induction and recursive definitions.
4.3, 4.4, 4.5: Euclidian algorithm, Fundamental Theorem of Arithmetic.
8.1: Statement and proof of the inclusion-exclusion principle.
8.3: Derangements, as an application of the inclusion-exclusion principle.

Midterm 2 Exam Syllabus: The focus will be on the following sections:

5.1, 5.2: Cartesian products, functions, injections
5.3: Surjections, Stirling numbers of the second kind.
5.5: Pigeonhole principle.
5.6: Bijections, composition of functions, inverse functions.
5.7: Computational complexity.
7.1, 7.2: Relations on a set, incidence matrices.
7.3: Partial orderings, Hasse diagrams.
7.4: Equivalence relations.

Syllabus

Week number: Monday date: Subtopics. Section numbers.

1: 29 Jan. (First class on Tuesday.) General outline of discrete mathematics. Counting arrangements of coloured balls in coloured boxes. Permutations 1.2. Enumerating subsets. Binomial coefficients, Pascal's Triangle, the Binomial Theorem, 1.3.

2: 5 Feb. Counting arrangements of plain balls in coloured boxes. Counting solutions to integer equations 1.3. Catalan numbers, 1.5.

3: 12 Feb. Simple mathematical induction, 4.1. Recursive definitions. Mathematical induction with various kinds of inductive hypothesis, 4.2. Euclidian algorithm 4.3. Fundamental Theorem of Arithmetic 4.4.

4: 19 Feb. Statement and proof of Inclusion-Exclusion Principle, 8.1. Illustration involving Euler's totient function. Derangements, use of inclusion-exclusion to prove enumerative formula for derangements, 8.3.

5: 26 Feb. Relations 5.1. Functions, bijections, injections, surjections. Composites of functions. Identity function. 5.2, 5.3, 5.6.

6: 5 Mar. Review for Midterm. (Midterm 1 on Tuesday 6th March.) Pigeonhole principle, 5.5. Introductory comments on Stirling numbers of the second kind.

7: 12 Mar. [Midterm 1 postmortem.] Equivalence relations. Counting equivalence relations and counting surjections using Stirling numbers of the second kind. Introduction to partial orderings

8: 19 Mar. (No class on Friday.) Partial orderings and Hasse diagrams.

9: 26 Mar. Enumeration of relations using incidence matrices. Pigeonhole Principle.

10: 2 Apr. Computational complexity. Linear homogeneous and inhomogeneous recurrence relations of first and second degree.

11: 9 Apr. Review for Midterm. (Midterm 2 on Tuesday 10th April.)

12: 16 Apr. [Midterm 2 postmortem.]

13: 23 Apr.

14: 30 Apr.

15: 7 May.

16: 14 May. (Last class on Friday.)