Quizzes, with solutions

MATH 225, *Linear algebra and differential equations*

Fall 2026, Sections 3 and 6

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General advice:

Show your working clearly. Present your conclusion clearly.

Marks may be lost if your writing is too scrawled or too faint or otherwise hard to read.

Dates:

3.1: Friday 9 October
6.1: Thursday 8 October
3.2: Friday 23 October
6.2: Monday 19 October

Quiz questions

Quiz 3.1: Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

Quiz 6.1: Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 2 & 1 \end{bmatrix}$.

Quiz 3.2: Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & 4 & 9 & 8 & 7 \\ 2 & 3 & 9 & 8 & 7 \\ 3 & 2 & 9 & 8 & 7 \\ 4 & 1 & 9 & 8 & 7 \\ 3 & 2 & 9 & 8 & 7 \\ 2 & 3 & 9 & 8 & 7 \\ 1 & 4 & 9 & 8 & 7 \end{bmatrix}$.

Quiz 6.2: Find the rank and nullity of the matrix $A = \begin{bmatrix} 3 & 2 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 3 & 2 \\ 5 & 6 & 7 & 10 & 9 & 8 \\ 3 & 2 & 1 & 2 & 3 & 4 \end{bmatrix}$.

Quiz solutions

Solution 3.1: Setting up the problem, $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$.

Subtracting twice row 1 from row 3 gives $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & -2 & 0 & 1 \end{array}\right]$.

Scaling two rows gives $\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 1/3 \end{array}\right]$.

Subtracting twice row 3 from row 1, we arrive at $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 0 & 2/3 \\ 0 & 1 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1 & -2/3 & 0 & 1/3 \end{array}\right]$.

In conclusion, $A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$.

Alternative solution: Since $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$, we have $A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{bmatrix}$.

Comment: It is easy to see that, given a $p \times p$ matrix P and a $q \times q$-matrix q then, as an equality of $(p+q) \times (p+q)$ matrices

$$\begin{bmatrix} P & 0_{p,q} \\ 0_{q,p} & Q \end{bmatrix}^{-1} = \begin{bmatrix} P^{-1} & 0_{p,q} \\ 0_{q,p} & Q^{-1} \end{bmatrix}$$

where $0_{p,q}$ denotes the zero $p \times q$ matrix. The alternative solution is an adaptation of that observation, reducible to that observation by renumbering the coordinates.

Solution 6.1: Starting from $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$,

subtracting multiples of the first row from the others to obtain $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & -2 & 1 & 0 \\ 0 & 2 & 1 & -3 & 0 & 1 \end{array}\right]$,

then scaling row 2 to get $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2/3 & 1/3 & 0 \\ 0 & 2 & 1 & -3 & 0 & 1 \end{array}\right]$,

and finally subtracting twice row 2 from row 3, which gives us $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & -2/3 & 1/3 & 0 \\ 0 & 2 & 1 & -5/3 & -2/3 & 1 \end{array}\right]$,

we conclude that $A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -2 & 1 & 0 \\ -5 & -2 & 3 \end{bmatrix}$.

Alternative solution: Inverses being unique, a vindicated trial solution will suffice. We posit

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ x & 1/3 & 0 \\ y & z & 1 \end{bmatrix}$$

for some $x, y, z \in \mathbb{R}$. By considering the 3 matrix entries below the leading diagonal in the equality $AA^{-1} = I$, we find that

$$2 + 3x = 3 + 2x + y = 2/3 + z = 0 \, .$$

Evidently, $x = -2/3$ and $y = -5/3$ and $z = -2/3$. In conclusion,

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1/3 & 0 \\ -5/3 & -2/3 & 1 \end{bmatrix} \, .$$

Comment: It is not hard to see that, for any invertible lower triangular matrix, the inverse is lower triangular. A similar comment holds for invertible upper triangular matrices. So the alternative method can be adapted to any invertible lower triangular matrix or any invertible upper triangular matrix. For such matrices, the method is very efficient. Essentially, it is Gaussian elimination with the elimination already complete at the start.

Solution 3.2: We have $\text{rank}(A) = 2$ and $\text{null}(A) = 3$. To see this, observe that columns 3, 4, 5 are all scalar multiples of the sum of columns 1 and 2. Plainly, columns 1 and 2 are linearly independent. So the rank is 2. By the rank-nullity formula $\text{rank}(A) + \text{null}(A) = 5$, the nullity is 3.

Solution 6.2: We have $\text{rank}(A) = 2$ and $\text{null}(A) = 4$.

Standard way: First subtract row 1 from row 4 so that the new row 4 becomes the zero row. Then swap rows 1 and 2. Then subtract multiples of row 1 from rows 2 and 3. Then divide rows 2 and 3 by -4. We arrive at the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 3 & 2 \\ 0 & 2 & 4 & 5 & 3 & 1 \\ 0 & 2 & 4 & 5 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \, .$$

Finally, subtract row 2 from row 3. It is now clear that the rank is 2. To complete the argument, we apply the rank-nullity formula, $\text{rank}(A) + \text{null}(A) = 6$.

Comment: Actually, it is probably a bad idea to do so many row operations all at once. That might invite mistakes.

Swifter way: Row 3 is row 1 plus twice row 2. Row 4 is a copy of row 1. So the row space is spanned by the first two rows which, clearly, are linearly independent. So the rank is as stated. Applying the rank-nullity formula as before, we deduce that the nullity is as stated too.