MATH 524, Algebra 2
Homeworks and Quizzes, Spring 2015

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version: 2 April 2015.

Office Hours: Mondays, 16:40 - 17:30, SAZ 129.

Office Hours would be a good time to ask me for help with the homeworks.

Homework 1 due Thursday 26th February

Review the proofs of the four results below, which concern the polynomial ring $F[X]$, where F is a field.

Proposition 1: Show that $F[X]$ is a Euclidian domain with respect to degree. In other words, given nonzero polynomials f and g then there exist polynomials q and r such that $f = qg + r$ and either $r = 0$ or else $\deg(r) \leq \deg(g)$.

Proposition 2: Show that, for any nonzero polynomials $f, g \in F[X]$, there exist polynomials $x, y \in F[X]$ such that $xf + yg$ divides f and g. Also show that $xf + yg$ is unique up to a non-zero factor in F. (We call $xf + yg$ the greatest common divisor of f and g.)

Proposition 3: Show that $F[X]$ is a unique factorization domain. In other words, any polynomial $f \in F[X]$ can be expressed in the form $af_1...f_r$ where $a \in F$ and f_1, ..., f_r are irreducible monic polynomials, and moreover, the factorization is unique in that, if $f = bg_1...g_s$ similarly, then $a = b$ and $r = s$ and, after renumbering, each $f_j = g_j$.

Proposition 4: Show that $F[X]$ is a principal ideal domain. In other words, every ideal of $F[X]$ can be expressed in the form $(f) = \{gf : g \in F[X]$ for some $f \in F[X]$.

Homework 2 due Thursday 16th April

2.1: Let Δ be a division ring and ley n be a positive integer. Show that $Z(\mathrm{Mat}_n(D)) \cong Z(D)$.

2.2: Is every subring of a semisimple ring semisimple?

2.3: Describe the semisimple $\mathbb{Z}$-modules.

2.4: Let R be the ring of continuous functions $[0, 1] \to \mathbb{R}$. Show that R is not semisimple.

2.5: Give a example of a finite-dimensional algebra A over $\mathbb{C}$ such that A is not semisimple.

2.6: Let R be a semisimple ring. Describe the two-sided ideals of R. Hence show that any quotient ring of R is semisimple.

2.7: Let M be a finitely-generated semsimple module of a ring R. Show that the ring $\mathrm{End}_R(M)$ is semisimple.

2.8: Up to isomorphism, how many 10-dimensional semisimple algebras over $\mathbb{C}$ are there?