Review the proofs of the four results below, which concern the polynomial ring $F[X]$, where $F$ is a field.

**Proposition 1:** Show that $F[X]$ is a Euclidian domain with respect to degree. In other words, given nonzero polynomials $f$ and $g$ then there exist polynomials $q$ and $r$ such that $f = qg + r$ and either $r = 0$ or else $\deg(r) \leq \deg(g)$.

**Proposition 2:** Show that, for any nonzero polynomials $f, g \in F[X]$, there exist polynomials $x, y \in F[X]$ such that $xf + yg$ divides $f$ and $g$. Also show that $xf + yg$ is unique up to a non-zero factor in $F$. (We call $xf + yg$ the greatest common divisor of $f$ and $g$.)

**Proposition 3:** Show that $F[X]$ is a unique factorization domain. In other words, any polynomial $f \in F[X]$ can be expressed in the form $af_1...f_r$ where $a \in F$ and $f_1, ..., f_r$ are irreducible monic polynomials, and moreover, the factorization is unique in that, if $f = bg_1...g_s$ similarly, then $a = b$ and $r = s$ and, after renumbering, each $f_j = g_j$.

**Proposition 4:** Show that $F[X]$ is a principal ideal domain. In other words, every ideal of $F[X]$ can be expressed in the form $(f) = \{gf : g \in F[X]\}$ for some $f \in F[X]$.

**Homework 2 due Thursday 16th April**

2.1: Let $\Delta$ be a division ring and let $n$ be a positive integer. Show that $Z(\text{Mat}_n(D)) \cong Z(D)$.

2.2: Is every subring of a semisimple ring semisimple?

2.3: Describe the semisimple $Z$-modules.

2.4: Let $R$ be the ring of continuous functions $[0, 1] \to \mathbb{R}$. Show that $R$ is not semisimple.

2.5: Give a example of a finite-dimensional algebra $A$ over $\mathbb{C}$ such that $A$ is not semisimple.

2.6: Let $R$ be a semisimple ring. Describe the two-sided ideals of $R$. Hence show that any quotient ring of $R$ is semisimple.

2.7: Let $M$ be a finitely-generated semisimple module of a ring $R$. Show that the ring $\text{End}_R(M)$ is semisimple.

2.8: Up to isomorphism, how many 10-dimensional semisimple algebras over $\mathbb{C}$ are there?