

Math.240,QUIZ 2**Surname&Name:****Question:** Consider the IVP

$$x \frac{dy}{dx} = \sqrt{1-y^2}, y(-1)=1$$

- What can you say about the existence and uniqueness of its solution?
- Is the given differential equation **separable**?
- Solve the given IVP.
- Determine the interval in which the solution(s) exist.

Answer: a) $f(x,y) = \frac{\sqrt{1-y^2}}{x}$ and $f_y = -\frac{y}{x\sqrt{1-y^2}}$ cannot be continuous in any rectangle around the point

$(-1,1)$. Hence we cannot use the Existence and Uniqueness Thm. to determine whether the given problem does or does not have a unique soln.

b) Yes.

$$c) \frac{1}{\sqrt{1-y^2}} dy = \frac{1}{x} dx, |y| < 1 \text{ and } x \neq 0$$

$$\Rightarrow \arcsin y = \ln|x| + C, C \in \mathbb{R}$$

$$\Rightarrow y = \sin(\ln|x| + C) \text{ if } |y| < 1 \text{ and } x \neq 0$$

Using the IC $y(-1)=1$, $y = \sin(\ln|x| + \frac{\pi}{2})$.

On the other hand, the constant fcn. $y=1$ satisfies the IVP and this soln. was lost during the separation process. Thus,

$y = \sin(\ln|x| + \frac{\pi}{2})$ and $y=1$ are the solns. of the given IVP.

d) $I = (-\infty, 0)$ which contains the initial point $x=-1$.