

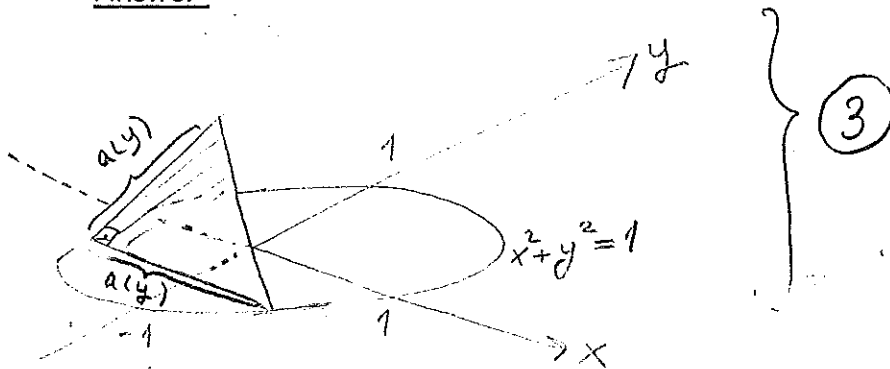
Surname: Key
Name:

Math.112(06) 2 (01)

Quiz 1

Question: The base of the solid is the disk $x^2 + y^2 \leq 1$. The cross-sections by planes perpendicular to the y -axis between $y = -1$ and $y = 1$ are isosceles right triangles with one leg in the disk. Find the volume of the solid.

Answer:



① $A(y) = \frac{1}{2} (a(y))^2 \rightarrow$ cross-sectional area

③ $\left\{ \begin{aligned} &= \frac{1}{2} \left[\sqrt{1-y^2} - (-\sqrt{1-y^2}) \right]^2 \\ &= 2(1-y^2) \end{aligned} \right.$

③ $\left\{ \begin{aligned} V &= \int_{-1}^1 2(1-y^2) dy = 4 \int_0^1 (1-y^2) dy = 4 \left(y - \frac{y^3}{3} \right) \Big|_{y=0}^1 = \frac{8}{3} \end{aligned} \right.$
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Name :

Spring 2009 - Math 112

Quiz 1

Find the volume of the solid whose cross-sections perpendicular to the x -axis are equilateral triangles whose heights run from the bottom of the region bounded by $y = 2 - x^2$, $y = x^3$, and x -axis to the top.

$$x^3 = 2 - x^2$$

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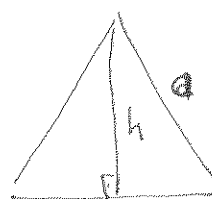
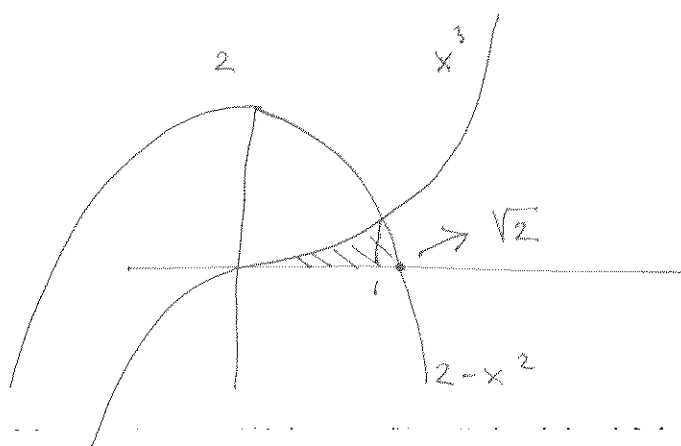
$$x^3 + x^2 - 2 = 0$$

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$$(x-1)(x^2 + 2x + 2) = 0$$

$\{$

$$x = 1$$



$$\frac{a\sqrt{3}}{2} = h \iff a = \frac{2h}{\sqrt{3}}$$

$$\text{Area} = \frac{ah}{2} = \frac{h^2}{\sqrt{3}}$$

$$\text{Volume} = \int_0^1 \frac{(x^3)^2}{\sqrt{3}} dx + \int_1^{\sqrt{2}} \frac{(2-x^2)^2}{\sqrt{3}} dx$$

$$= \frac{1}{\sqrt{3}} \left[\frac{x^7}{7} \Big|_0^1 + \left(4x - \frac{2}{3}x^3 + \frac{x^5}{5} \right) \Big|_1^{\sqrt{2}} \right]$$

Question: Let $f(x)$ be a positive continuous function on $[1, 4]$. Let R_1 be the region bounded by the curves $y = f(x^2)$, $x = 1$, $x = 2$, and $y = 0$. Let R_2 be the region bounded by the curves $y = \sqrt{f(x)}$, $x = 1$, $x = 4$, and $y = 0$. Show that the volume of the solid S_1 obtained by rotating the region R_1 about the y -axis is equal to the volume of the solid S_2 obtained by rotating the region R_2 about the x -axis.

Solution: The volume of S_1 is equal by the shell method to the integral

$$V_1 = \int_1^2 2\pi x f(x^2) dx,$$

and the volume of the solid S_2 is equal by the disc method to the integral

$$V_2 = \int_1^4 \pi (\sqrt{f(x)})^2 dx = \int_1^4 \pi f(x) dx.$$

We are required to show that the integrals V_1 and V_2 are equal. Indeed, the substitution

$$u = x^2$$

transforms the integral V_1 to the integral V_2 . To be more explicit,

$$V_1 = \int_1^2 2\pi x f(x^2) dx \quad \underbrace{=}_{u=x^2, du=2x dx} \int_1^4 \pi f(u) du = \int_1^4 \pi f(x) dx = V_2.$$

Writing V_1 worths 4 points.

Writing V_2 worths 4 points.

Showing the equality of V_1 and V_2 worths 2 points.

Question: Find the length of the curve

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$$\begin{cases} x = t^2 + \frac{1}{2t} \\ y = 4\sqrt{t} \end{cases}, \quad \frac{1}{\sqrt{2}} \leq t \leq 1$$

Sol:

$$\frac{dx}{dt} = 2t - \frac{1}{2t^2}$$

$$\frac{dy}{dt} = \frac{2}{\sqrt{t}}$$

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= 4t^2 - 2\frac{1}{t} + \frac{1}{4t^4} + \frac{4}{t} \\ &= 4t^2 + 2\frac{1}{t} + \frac{1}{4t^4} \\ &= \left(2t + \frac{1}{2t^2}\right)^2 \end{aligned}$$

$$L = \int_{1/\sqrt{2}}^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_{1/\sqrt{2}}^1 \left(2t + \frac{1}{2t^2}\right) dt$$

$$= \left(t^2 - \frac{1}{2t}\right) \Big|_{1/\sqrt{2}}^1 = \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}$$

Quiz 3

474/21 Find the ^{surface} area of the surface generated by revolving the curve

$$C: x = \frac{e^y + e^{-y}}{2}, \quad 0 \leq y \leq \ln 2$$

about the y -axis.

$$S = 2\pi \int_0^{\ln 2} x \, ds \quad \text{where} \quad ds = \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{\left(\frac{dx}{dy} dy\right)^2 + (dy)^2}$$

$$= \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} \, dy$$

$$\frac{dx}{dy} = \frac{e^y - e^{-y}}{2}, \quad \sqrt{1 + \left(\frac{dx}{dy}\right)^2} = \sqrt{1 + \left(\frac{e^y - e^{-y}}{2}\right)^2}$$

$$= \sqrt{\frac{e^{2y} + 2 + e^{-2y}}{4}}$$

$$= \frac{e^y + e^{-y}}{2}$$

Then

$$S = 2\pi \int_0^{\ln 2} \left(\frac{e^y + e^{-y}}{2}\right)^2 dy$$

$$= \frac{\pi}{2} \int_0^{\ln 2} (e^{2y} + 2 + e^{-2y}) dy$$

$$= \frac{\pi}{2} \left(\frac{e^{2y}}{2} + 2y - \frac{e^{-2y}}{2} \right) \Big|_{y=0}^{\ln 2}$$

$$= \pi \left(\frac{15}{16} + \ln 2 \right)$$

Quiz 4

Q) Show that $\tanh^{-1}x = \frac{1}{2} \ln \frac{1+x}{1-x}$, $|x| < 1$

Answer: Let $y = \tanh^{-1}x$. Then $x = \tanh y$, or
(1) $x = \frac{e^y - e^{-y}}{e^y + e^{-y}} \dots (A)$

Let's solve (A) for y .

$$(A) \Rightarrow x e^y + x e^{-y} = e^y - e^{-y} \quad (1)$$

$$(1) \quad \frac{e^{-y}}{0, \neq y} [x e^{2y} + x - e^{2y} + 1] = 0 \quad (1)$$

$$\Rightarrow e^{2y}(x-1) = -(1+x)$$

$$\text{or } e^{2y} = \frac{1+x}{1-x} \quad (1)$$

$$\text{Then } e^y = \pm \sqrt{\frac{1+x}{1-x}} \quad (1)$$

$$\text{Since } e^y > 0, \quad e^y = \sqrt{\frac{1+x}{1-x}}, \quad |x| < 1. \quad (1)$$

$$\text{Hence, } y = \ln \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}}, \text{ or equivalently } (1)$$

$$\tanh^{-1}x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right), \quad |x| < 1.$$

(1)

Quiz 5

Find a fcn. f , cont. $\forall x$ (and not everywhere zero), s.t.

$$8 - \frac{f^2(x)}{2} = \int_0^x \frac{f^2(t) \sin t}{2 + \cos t} dt \dots (1)$$

$$-ff' = \frac{f^2 \sin x}{2 + \cos x} \quad (2)$$

$$(2) \quad \frac{f'}{f} = -\frac{\sin x}{2 + \cos x}, \quad f \neq 0$$

$$(2) \quad \ln|f| = \ln(2 + \cos x) + C_1 \quad \text{where} \quad \int \frac{-\sin x}{2 + \cos x} dx = \ln(2 + \cos x) + C$$

$$(1) \quad |f(x)| = C_1 (2 + \cos x)$$

$$(1) \Rightarrow 8 - \frac{f^2(0)}{2} = 0 \Rightarrow f(0) = \pm 4 \quad (1)$$

$$f(0) = 3C = \pm 4 \Rightarrow C = \pm \frac{4}{3} \quad (1)$$

$$f(x) = \pm \frac{4}{3} (2 + \cos x) \quad (1)$$

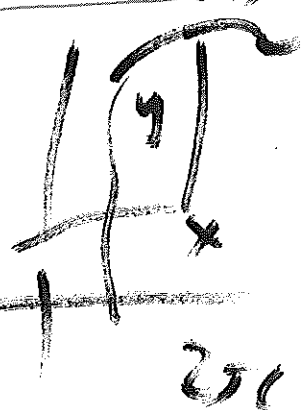
Find the area of the surface generated by revolving the curve $\begin{cases} x = \frac{t^2}{2} + t \\ y = 2t \end{cases}$ about the line $y = -2$ for $\sqrt{5}-1 \leq t \leq \sqrt{12}-1$.

Solution

$$S = \int_{\sqrt{5}-1}^{\sqrt{12}-1} 2\pi (y+2) ds = \int_{\sqrt{5}-1}^{\sqrt{12}-1} 2\pi (y+2) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$\frac{dx}{dt} = t+1$$

$$\frac{dy}{dt} = 2$$



$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^2 + 2t + 5$$

$$S = \int_{\sqrt{5}-1}^{\sqrt{12}-1} 2\pi (2t+2) \sqrt{t^2+2t+5} dt$$

$$= \int_4^9 2\pi \sqrt{u} du = \frac{4\pi}{3} u^{3/2} \Big|_4^9$$

$$u = t^2 + 2t + 5$$

$$du = (2t+2)dt$$

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1

Question: let f be a continuous function on $[0, 1]$. show that

$$\int_0^1 \left(\int_0^x f(t) dt \right) dx = \int_0^1 (1-x) f(x) dx$$

Solution:

$$\int_0^1 \left(\int_0^x f(t) dt \right) dx = \left(x \int_0^x f(t) dt \right) \Big|_0^1 - \int_0^1 x f(x) dx \quad (1)$$

$$\begin{aligned} (1) \quad & \boxed{u = \int_0^x f(t) dt} \Rightarrow \boxed{du = f(x) dx} \quad (2) \\ & \boxed{dv = dx} \Rightarrow \boxed{v = x} \quad (1) \end{aligned}$$

$$= \left[1 \int_0^1 f(t) dt - 0 \int_0^0 f(t) dt \right] - \int_0^1 x f(x) dx \quad (2)$$

$$= \int_0^1 f(t) dt - \int_0^1 x f(x) dx$$

$$\begin{aligned} (2) \quad & \xrightarrow{\parallel} \\ & = \int_0^1 f(x) dx - \int_0^1 x f(x) dx \\ & = \int_0^1 (1-x) f(x) dx \quad (2) \end{aligned}$$

Math 112; Section 01 (Erqun)

Quiz 4

Section 01, Engin, Quiz

Question $\int \ln(x^2+x+2) dx = ?$

$$\int \ln(x^2+x+2) dx = x \ln(x^2+x+2) - \int \frac{2x^2+x}{x^2+x+2} dx \quad (1)$$

$$(1) \left[\begin{array}{l} u = \ln(x^2+x+2), du = \frac{2x+1}{x^2+x+2} \\ dv = dx, v = x \end{array} \right] \quad \text{!!}$$

$$(1) \left[\frac{2x^2+x}{x^2+x+2} = 2 - \frac{x+4}{x^2+x+2} \right] = 2 - \frac{x+(1/2)}{x^2+x+2} - \frac{7/2}{x^2+x+2} \quad (2)$$

$$\text{As } (x^2+x+2)' = 2x+1 = 2(x+\frac{1}{2})$$

$$\text{Thus, } I = \int \frac{2x^2+x}{x^2+x+2} = \int 2 dx - \int \frac{x+(1/2)}{x^2+x+2} dx - \frac{7}{2} \int \frac{dx}{x^2+x+2} \quad (1)$$

$$= 2x - \frac{1}{2} \ln(x^2+x+2) - \frac{7}{2} \int \frac{dx}{x^2+x+2}$$

$$(2) \left[x^2+x+2 = (x+\frac{1}{2})^2 + \frac{7}{4} = \frac{7}{4} \left(\left(\sqrt{\frac{4}{7}}(x+\frac{1}{2}) \right)^2 + 1 \right) \right] \quad \text{!!}$$

$$J = \int \frac{dx}{x^2+x+2} = \frac{4}{7} \int \frac{dx}{\left(\sqrt{\frac{4}{7}}(x+\frac{1}{2}) \right)^2 + 1} = \frac{4}{7} \sqrt{\frac{4}{7}} \tan^{-1} \left(\sqrt{\frac{7}{4}}(x+\frac{1}{2}) \right) + C \quad (1)$$

Therefore,

$$\int \ln(x^2+x+2) dx = x \ln(x^2+x+2) - 2x + \frac{1}{2} \ln(x^2+x+2) + \frac{7}{2} \frac{4}{7} \sqrt{\frac{4}{7}} \tan^{-1} \left(\sqrt{\frac{7}{4}}(x+\frac{1}{2}) \right) + C'$$

(1)

Question $\int \cos^{-1}(\sqrt{x}) dx = ?$

Section 03

Q5

Write down the partial fractional decomposition for

$$(2x^2 + x - 6)(x^4 + 2x^3)$$

$$(2x^2 - 3x + 10)(3x^2 + 5x - 2)(3x - 1)$$

Factorization & Cancellation:

$$\frac{(2x-3)(x+2)^2 x^3}{(2x^2-3x+10)(3x-1)^2(x+2)}$$

} 4 → 2 pt for cancelling $x+2$
→ 2 pt for correct factorization

Decomposition:

$$\frac{Ax+B}{2x^2-3x+10} + \frac{C}{3x-1} + \frac{D}{(3x-1)^2}$$

(2) (2) (2)

Quiz 6

Evaluate $I = \int x^2 \tan^{-1} x \, dx$

$$u = \tan^{-1} x, \, dv = x^2 \, dx$$

$$du = \frac{dx}{1+x^2}, \, v = \frac{x^3}{3}$$

$$I = \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \frac{x^3}{1+x^2} \, dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{2x}{2(x^2+1)} \right) \, dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \left(\frac{x^2}{2} - \frac{1}{2} \ln(x^2+1) \right) + C$$

$$\begin{array}{r} x^3 \overline{) x^2+1} \\ \underline{-x^3+x} \quad x \\ -x \end{array}$$

Question: $I = \int \frac{dx}{(3x^2 - 12x + 8)^{5/2}} = ?$

Math 112 Sec 03

Sol: $3x^2 - 12x + 8 = 3(x^2 - 4x) + 8 = 3((x-2)^2 - 4) + 8 = 3(x-2)^2 - 4$ (3)

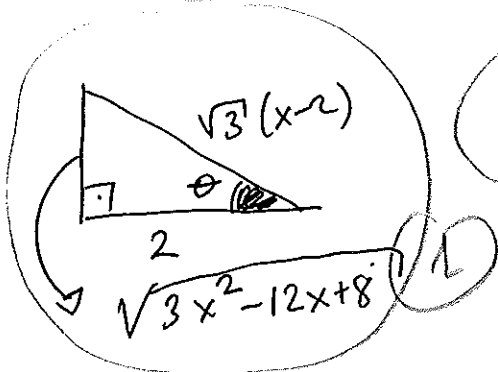
Put $\sqrt{3}(x-2) = 2\sec\theta$ (2) Then $\sqrt{3}dx = 2\sec\theta \tan\theta d\theta$ (1)
and $3x^2 - 12x + 8 = 3(x-2)^2 - 4 = 4(\sec^2\theta - 1) = 4\tan^2\theta$ (1) So

$I = \int \frac{(2/\sqrt{3}) \sec\theta \tan\theta d\theta}{2^5 \tan^5\theta} = \frac{1}{16\sqrt{3}} \int \frac{\sec\theta}{\tan^4\theta} d\theta$ (1)

$= \frac{1}{16\sqrt{3}} \int \frac{\cos^3\theta}{\sin^4\theta} d\theta = \frac{1}{16\sqrt{3}} \int \frac{(1-\sin^2\theta)\cos\theta}{\sin^4\theta} d\theta = \frac{1}{16\sqrt{3}} \int \frac{1-u^2}{u^4} du$ (1)

$= \frac{1}{16\sqrt{3}} \int \left(\frac{1}{u^4} - \frac{1}{u^2} \right) du = \frac{1}{16\sqrt{3}} \left(-\frac{1}{3} \frac{1}{u^3} + \frac{1}{u} \right) + C$ (1)

$= \frac{-1}{48\sqrt{3}} \frac{1}{\sin^3\theta} + \frac{1}{16\sqrt{3}} \frac{1}{\sin\theta} + C$ (1)



$I = \frac{-1}{48\sqrt{3}} \left(\frac{\sqrt{3}(x-2)}{\sqrt{3x^2 - 12x + 8}} \right)^3 + \frac{1}{16\sqrt{3}} \left(\frac{\sqrt{3}(x-2)}{\sqrt{3x^2 - 12x + 8}} \right) + C$ (1)

$= \frac{-1}{16} \frac{(x-2)^3}{(3x^2 - 12x + 8)^{3/2}} + \frac{1}{16} \frac{(x-2)}{(3x^2 - 12x + 8)^{1/2}} + C$

~~If one gets more than 10 points
his/her grade will be 10~~

Question: $I = \int \frac{x dx}{(9x^2 - 36x + 32)^{3/2}} = ?$

Sec 01
Ex 1

Sol As $\frac{d}{dx} (9x^2 - 36x + 32) = 18x - 36 = 18(x-2)$,

$$I = \underbrace{\int \frac{(x-2) dx}{(9x^2 - 36x + 32)^{3/2}}}_{I_1} + \underbrace{\int \frac{2 dx}{(9x^2 - 36x + 32)^{3/2}}}_{I_2} \quad (1)$$

$$I_1 = \frac{1}{18} \int \frac{du}{u^{3/2}} = \frac{1}{18} (-2) u^{-1/2} + C_1 = \frac{-1}{9(9x^2 - 36x + 32)^{1/2}} + C_1$$

$u = 9x^2 - 36x + 32$
 $du = 18(x-2) dx$

$$9x^2 - 36x + 32 = 9(x^2 - 4x) + 32 = 9(x^2 - 4x + 4) + 32 = 9(x-2)^2 - 4$$

Put $3(x-2) = 2 \sec \theta$ Then, $3 dx = 2 \sec \theta \tan \theta d\theta$, and

$9x^2 - 36x + 32 = 4 \tan^2 \theta$. So, I_2 becomes

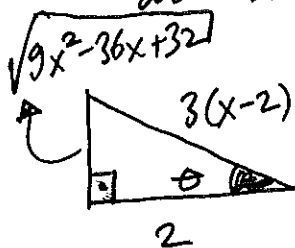
$$I_2 = \int \frac{2(2/3) \sec \theta \tan \theta d\theta}{2^3 \tan^3 \theta} = \frac{1}{6} \int \frac{\sec \theta}{\tan^2 \theta} d\theta = \frac{1}{6} \int \frac{\cos \theta}{\sin^2 \theta} d\theta$$

$$= \frac{1}{6} \int \frac{dt}{t^2} = -\frac{1}{6} \frac{1}{t} + C_2 = \frac{-1}{6 \sin \theta} + C_2$$

$t = \sin \theta$

$dt = \cos \theta d\theta$

$$= \frac{-(x-2)}{2 \sqrt{9x^2 - 36x + 32}} + C_2$$



Consequently,

$$I = \frac{-1}{9 \sqrt{9x^2 - 36x + 32}} + \frac{-(x-2)}{2 \sqrt{9x^2 - 36x + 32}} + C$$

$$= \frac{16 - 9x}{18 \sqrt{9x^2 - 36x + 32}} + C$$

Quiz 7

Determine whether the integral $\int_1^\infty \frac{1}{\sqrt{e^x - x}} dx$ converges.

Note that

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{e^x - x}} = 1 \quad \text{and} \quad \int_1^\infty e^{-x/2} dx = -2e^{-x/2} \Big|_1^\infty = \frac{2}{\sqrt{e}}.$$

Hence by limit comparison theorem the first integral also converges.

Is the improper integral $\int_0^{\infty} \frac{dx}{(x+x^4)^{1/3}}$ convergent?

Solution:

$$\int_0^{\infty} \frac{dx}{(x+x^4)^{1/3}} = \underbrace{\int_0^1 \frac{dx}{(x+x^4)^{1/3}}}_{I''} + \underbrace{\int_1^{\infty} \frac{dx}{(x+x^4)^{1/3}}}_{J''} \quad (1)$$

$0 \leq \frac{1}{(x+x^4)^{1/3}} \leq \frac{1}{x^{4/3}} \quad (2)$ for $x \in (0, 1]$. As $\int_0^1 \frac{dx}{x^{4/3}}$ is convergent, (1) the direct comparison test implies that I is convergent. (1)

$0 \leq \frac{1}{(x+x^4)^{1/3}} \leq \frac{1}{x^{4/3}} \quad (2)$ for $x \in [1, \infty)$. As $\int_1^{\infty} \frac{dx}{x^{4/3}}$ is convergent, (1) the direct comparison test implies that J is convergent. (1)

Consequently, $\int_0^{\infty} \frac{dx}{(x+x^4)^{1/3}}$ is convergent. (1)

$$I = \int \frac{4 - (2x+3)^{1/2}}{(2x+3) + (2x+3)^{2/3}} dx = ?$$

Qus 2

Sol Let $u^6 = (2x+3)$ (2) Then $6u^5 du = 2dx$ (1) and

$$I = \int \frac{4 - u^3}{u^6 + u^4} 3u^5 du = 3 \int \frac{4 - u^3}{u^2 + 1} u du$$

$$= 3 \int \frac{-u^4 + 4u}{u^2 + 1} du \quad (2)$$

$$\frac{-u^4 + 4u}{-u^4 - u^2} \bigg| \frac{u^2 + 1}{-u^2 + 1}$$

$$\frac{u^2 + 4u}{u^2 + 1}$$

$$\frac{4u - 1}{u^2 + 1}$$

So $\frac{-u^4 + 4u}{u^2 + 1} = (-u^2 + 1) + \frac{4u - 1}{u^2 + 1}$ (2)

So, $I = 3 \int (-u^2 + 1 + \frac{4u}{u^2 + 1} - \frac{1}{u^2 + 1}) du$ (1)

$$= 3 \left(-\frac{u^3}{3} + u + 2 \ln(u^2 + 1) - \arctan u \right) + C \quad (2)$$

$$= -u^3 + 3u + 6 \ln(u^2 + 1) - 3 \arctan u + C$$

$$= -(2x+3)^{1/2} + 3(2x+3)^{1/6} + 6 \ln((2x+3)^{1/3} + 1)$$

$$- 3 \arctan((2x+3)^{1/6}) + C$$

— 0 —

Sec 01. Erquin

Quiz 8

Determine whether $\int_1^{\infty} \frac{e^x}{x} dx$ converges or not.

$$e^x > 1, x \geq 1 \Rightarrow \frac{e^x}{x} > \frac{1}{x}, x \geq 1$$

$$\text{and } \int_1^{\infty} \frac{dx}{x} \text{ diverges, } p=1$$

So, $\int_1^{\infty} \frac{e^x}{x} dx$ diverges by the DCT.

OR

$$\frac{e^x}{x} > \frac{x}{x} = 1 \text{ and } \int_1^{\infty} dx \text{ diverges} \xRightarrow{\text{DCT}} \int_1^{\infty} \frac{e^x}{x} dx \text{ diverges.}$$

OR, you can use LCT.

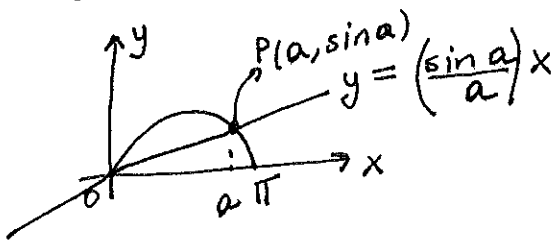
Quiz 8

Estimate the convergence behavior of

$$\int_0^{\infty} \frac{|\sin x|}{x^2} dx.$$

$$\int_0^{\infty} \frac{|\sin x|}{x^2} dx = \int_0^a \frac{|\sin x|}{x^2} dx + \int_a^{\infty} \frac{|\sin x|}{x^2} dx$$

where $a \in (0, \pi)$.



$$\sin x \geq \left(\frac{\sin a}{a}\right)x$$

$$\frac{|\sin x|}{x^2} = \frac{\sin x}{x^2} \geq \frac{\frac{\sin a}{a}}{x}, \quad x \in (0, a]$$

$$\neq \int_0^a \frac{\frac{\sin a}{a}}{x} dx \text{ div., } p=1$$

$$\stackrel{\text{DCT}}{\Rightarrow} \int_0^a \frac{\sin x}{x^2} dx \text{ diverges.}$$

$$\frac{|\sin x|}{x^2} \leq \frac{1}{x^2}, \quad x \in [a, \infty) \text{ and } \int_a^{\infty} \frac{dx}{x^2} \text{ conv., } p=2>1$$

$$\stackrel{\text{DCT}}{\Rightarrow} \int_a^{\infty} \frac{|\sin x|}{x^2} dx \text{ converges.}$$

$$\text{Hence, } \int_0^{\infty} \frac{|\sin x|}{x^2} dx \text{ diverges.}$$

QUIZ 8 Kopyası

Find exact value of

$$\sum_{n=0}^{\infty} (\arctan n - \arctan (n+2))$$

Let $f(n) = \arctan n$

$$\begin{aligned} S_n &= f(0) - \cancel{f(2)} + f(1) - \cancel{f(3)} \\ &\quad + \cancel{f(2)} - \cancel{f(4)} + \cancel{f(3)} - \cancel{f(5)} \\ &\quad + \dots \\ &\quad + \cancel{f(n-2)} - \cancel{f(n)} + \cancel{f(n-1)} - f(n+1) \\ &\quad + \cancel{f(n-1)} - f(n+2) \end{aligned}$$

$$\therefore S_n = f(0) + f(1) - f(n+1) - f(n+2)$$

$$\lim_{n \rightarrow \infty} S_n = 0 + \frac{\pi}{4} - \frac{\pi}{2} - \frac{\pi}{2}$$